Complex Networks (MTH6142)
Formative Assignment 2

- **1* Average degree of a growing network**

  Assume that you are observing a growing undirected network.
  The network evolves in time by the following rules:
  At time $t = 1$ there is a single isolated node.
  At each time $t > 1$ a new node is added to the network and is connected to the existing network by a new link.
  Consider the network at time $t = T$.

  (a) What is the total number of nodes N?

  (b) What is the total number of links L?

  (c) What is the average degree $\langle k \rangle$?

  (d) What is the average degree in the limit $T \to \infty$?

- **2. Matrix Formalism.**
  Consider a simple network of size N. Let $\mathbf{A}$ be the $N \times N$ adjacency matrix and let $\mathbf{1}$ be the N dimensional column vector whose elements are given by $1_i = 1 \; \forall i = 1, 2, \ldots N$, and $\mathbf{k}$ be the N dimensional column vector whose elements are given by the degrees $k_i \; \forall i = 1, 2, \ldots, N$, i.e.

  $$\mathbf{1} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}, \qquad \mathbf{k} = \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_N \end{pmatrix} \tag{1}$$

  Using the matrix formalism (row by column product) show that

  (a) the vector $\mathbf{k}$ whose elements are the degrees k_i of the nodes $i = 1, 2, \ldots, N$ can be written as

  $$\mathbf{k} = \mathbf{A1}. \tag{2}$$

  (b) the number L of links in the network can be written as

  $$L = \frac{1}{2}\mathbf{1}^T\mathbf{A1} \tag{3}$$

  (c) the matrix $\mathbf{N}$ whose element N_{ij} is equal to the number of common neighbours of nodes i and j can be written as

  $$\mathbf{N} = \mathbf{A^2} \tag{4}$$

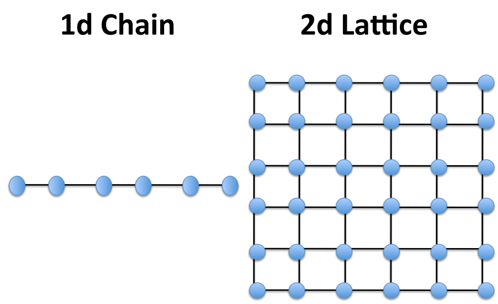

Figure 1: A linear chain network and a square portion of a square lattice with $l = 6$ nodes along each side.

- $3^\star$. **Diameter of simple networks.**
  One can calculate the diameter of certain types of network exactly. Assume that each of these network has network size N.

  (a) What is the diameter of a fully connected network?

  (b) What is the diameter of a star network?

  (c) What is the diameter of a linear chain of N nodes? (see figure 1)

  (d) What is the diameter D of a square portion of square lattice, with l nodes along each side (see figure 1) ?

  (e) Consider the expression found in question (3d) and find the leading term of D in terms of the total number of nodes N in the network, in the limit $N \gg 1$ proving that for $N \gg 1$

  $$D \simeq 2\sqrt{N}. \tag{5}$$

  (f) Which of the above networks have the small-world distance property?