WEEK 2 Tutorial

TAKE HOME MESSAGE FROM W1-W2

- Simple, unweighted/weighted, undirected/directed — NETWORKS + tadpoles + multiple links
- Bipartite
- REPRESENTATIONS: Edge list, Adjacency matrix A

N # of nodes

L # of links

$$k_i^{IN} = \sum_{j=1}^{N} A_{ij} \qquad k_i^{out} = \sum_{j=1}^{N} A_{ji}$$

node degrees

$$\{k_i\}_{i=1,2,\dots,N} = \{k_1, k_2, \dots, k_N\}$$

degree sequence

$$P(k) = \frac{N(k)}{N}$$ ← # of nodes of degree k

degree distribution

$$\langle k \rangle = \frac{1}{N} \sum_{i=1}^{N} k_i = \sum_k k P(k)$$

average (node) degree

$$K = \max_i k_i$$

maximum degree

Formative Assignment 1

Q1

- **1*. Adjacency matrix.**
  Consider the following adjacency matrix of a network

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix} \quad (1)$$

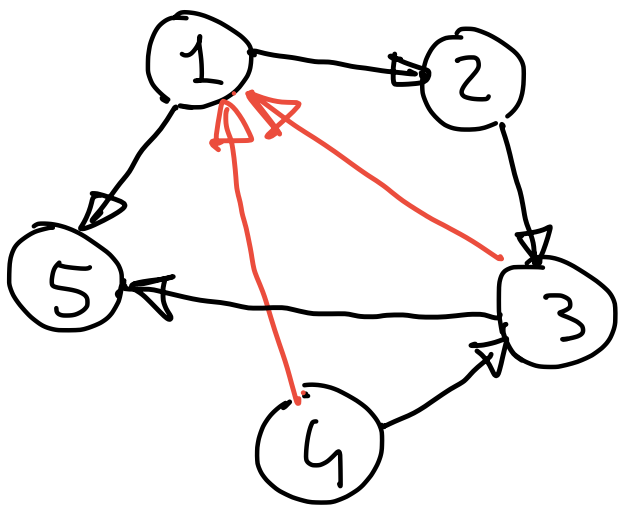
  a) Is the network directed or undirected? (*Explain why*).
  b) Draw the network.
  c) List the in-degree sequence and the out-degree sequence of the network
  d) Determine the in-degree distribution and the out-degree distribution.

(a) $N = 5$ nodes

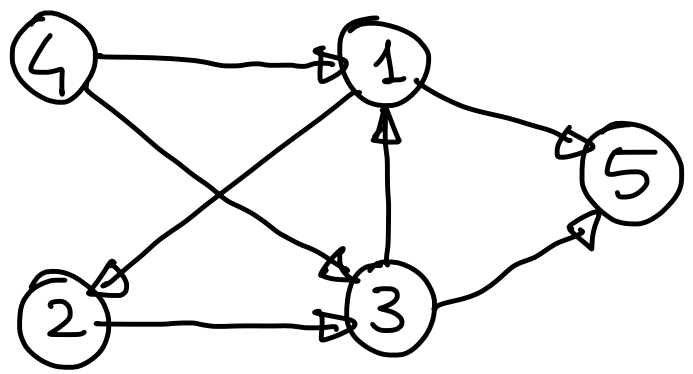
A is asymmetric $\Rightarrow$ the network is DIRECTED with no tadpoles

$$L = \sum_{i=1}^{5} \sum_{j=1}^{5} A_{ij} = 7 \quad \text{directed links}$$

(b)



Maybe this is a better drawing?



(c)

$$k_1^{IN} = \sum_{j=1}^{5} A_{1j} = 2 \qquad k_2^{IN} = 1 \qquad k_3^{IN} = 2 \qquad k_4^{IN} = 0 \qquad k_5^{IN} = 2$$

Summing over rows

In-degree sequence $\{2, 1, 2, 0, 2\}$

$$k_1^{OUT} = \sum_{J=1}^{5} A_{J1} = 2 \qquad k_2^{OUT} = 1 \qquad k_3^{OUT} = 2 \qquad k_4^{OUT} = 2 \qquad k_5^{OUT} = 0$$

Out-degree sequence $\{2, 1, 2, 2, 0\}$

(d)

$$P^{IN}(k) = \frac{N^{IN}(k)}{N}$$ ← 5

$$k = 0, 1, 2, 3, 4, 5$$

$$N^{IN}(k) = \sum_{i=1}^{5} \delta(k_i^{IN}, k)$$

$N^{IN}(0) = 1$ $\quad N^{IN}(1) = 1$ $\quad N^{IN}(2) = 3$ $\quad N^{IN}(k) = 0$ for $k = 3, 4, 5$

$P^{IN}(0) = \frac{1}{5}$ $\quad P^{IN}(1) = \frac{1}{5}$ $\quad P^{IN}(2) = \frac{3}{5}$ $\quad P^{IN}(k) = 0$ for $k = 3, 4, 5$

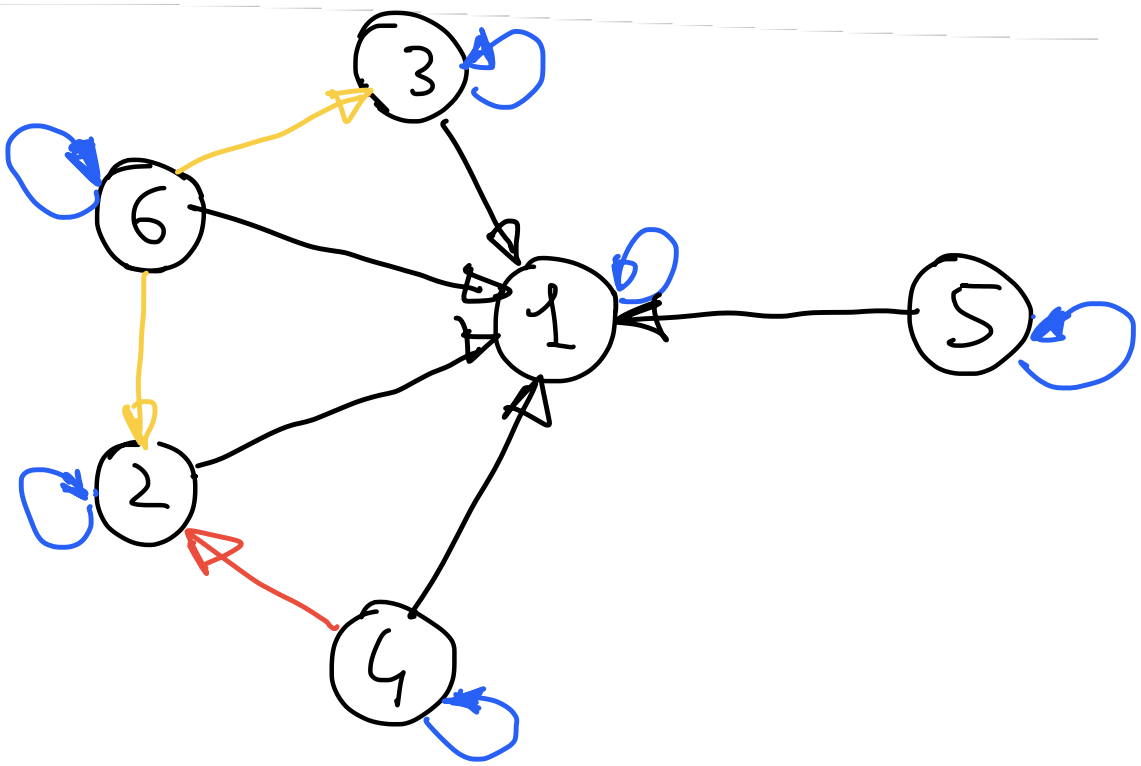
Analogously

$P^{OUT}(0) = \frac{1}{5}$ $\quad P^{OUT}(1) = \frac{1}{5}$ $\quad P^{OUT}(2) = \frac{3}{5}$ $\quad P^{OUT}(k) = 0$ for $k = 3, 4, 5$

Q2

- **2*. Number network.**
  Given the set of nodes V with $|V| = 6$ in which each node i is labelled by a natural number between 1 and 6, $i = 1, 2, 3, 4, 5, 6$, consider the directed network $G = (V, E)$ where each link from node j to node i indicates that j is a multiple of i.
  a) Draw the network.
  b) Write down the adjacency matrix of the network.
  c) Are there tadpoles in the network? How many?

@



(b)

$N = 6$

$$A = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \end{array} \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{matrix}$$

Yes we have 6 tadpoles

Q 3 Examples of complex networks

	NODES	LINKS	TYPE
HUMAN INTERACTIONS	People	Conversation	Undirected weights (# of meetings?)
AIR TRANSPORTATION	airports	Connections	directed weights (# of flights per day)

IMPORTANT: QUIZ 1 opens at 6PM today

↑ Assessed (4%)