Week 1 Lecture 2

Organization of the module

- CH 1 Introduction
- CH 2 Structural properties
- CH 3 Centrality
- CH 4 Random graphs
- CH 5 Scale-free networks
- CH 6 Evolving networks
- CH 7 Small-world networks
- CH 8 The configuration model

CHAPTER 1 INTRODUCTION

1.1 NETWORKS and GRAPHS

DEF GRAPHS

A graph $G=(V, E)$ consists of two sets V and E

The elements of V are called the vertices of the graph G

The elements of E are pairs of elements of V and one called edges

↑ ordered, or non ordered

Complex networks are the graphs that describe interactions in complex systems ← for a definition see lecture 1

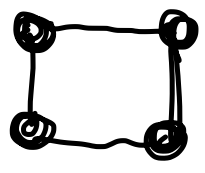
DEF COMPLEX NETWORKS

A complex network is the graph $G=(V, E)$ representing the set of interactions between the elements of a complex system

In complex networks the vertices are also called NODES and the edges are called LINKS

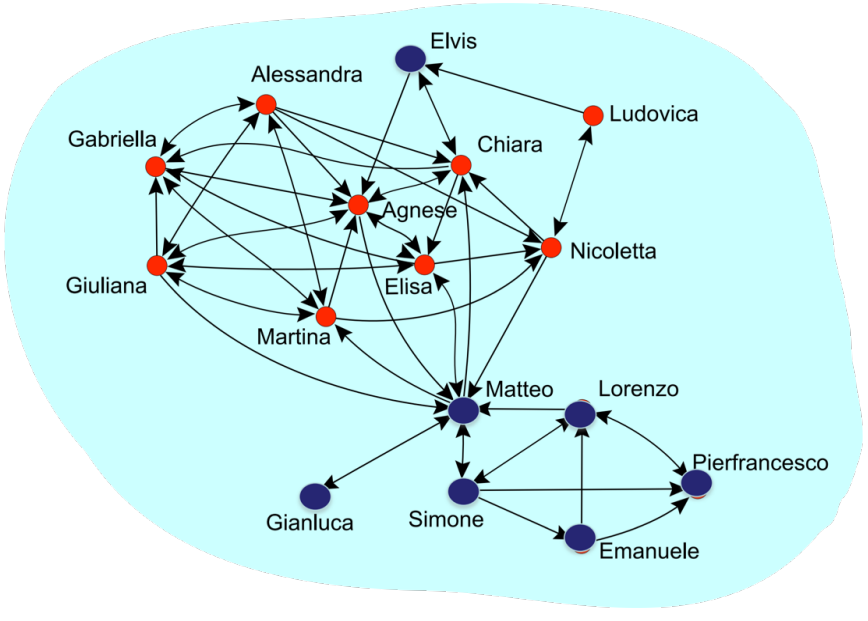
$N = |V|$ the # of nodes in the network ← ORDER (SIZE)

$L = |E|$ the # of links in the network

GRAPHS		NETWORKS
Vertices		Nodes
Edges		Links
Cycles		Loops
Loops		Tadpoles

1.2 EXAMPLES

See slides of Lecture 1



Elisa's kindergarten network

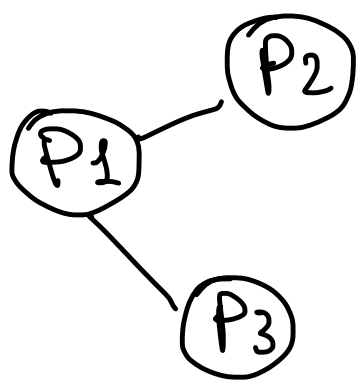
(a social network)

1.3 DIFFERENT TYPES OF NETWORKS

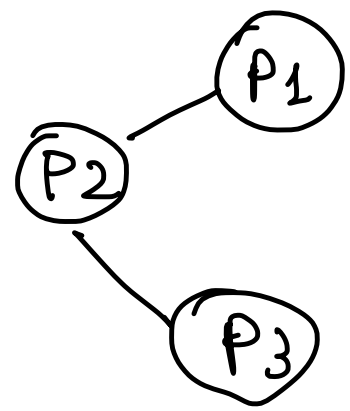
The nodes of a graph can be LABELLED or UNLABELLED

In networks usually nodes have a specific "NAME" or LABEL

EX In a protein interaction network each node is a specific protein



is not equivalent to



We will work with labelled networks

DEF LABELLED NETWORKS

A labelled network of size N is formed by a set V of N distinguishable nodes indicated by a different (unique) label $i = 1, 2, \dots, N$. The set E of L links is indicated as

$$E = \left\{ (i_1, j_1), (i_2, j_2), \dots, (i_L, j_L) \right\}$$

A labelled network can be DIRECTED or UNDIRECTED

DEF DIRECTED NETWORKS

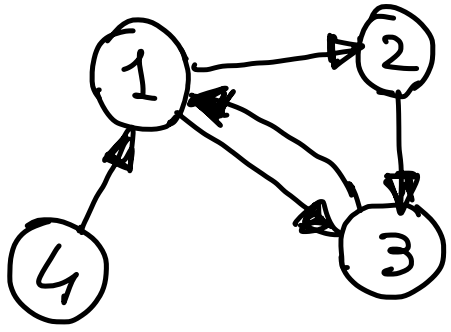
A directed link indicates an interaction between nodes that is NOT symmetrical, that is an ORDERED pair (i,j)

can be graphically represented as an arrow from i to j

"i points to j"

A directed network is a network in which every link is directed

EX



$N = 4$

$L = 5$

WWW, prey-predators

Facebook

DEF UNDIRECTED NETWORKS

An undirected link indicates a symmetric interaction, that is an UNORDERED pair (i,j) ← $(i,j) = (j,i)$

graphically represented as a line

"i and j are linked"

is equivalent to

"j and i are linked"

An undirected network is a network in which every link is undirected

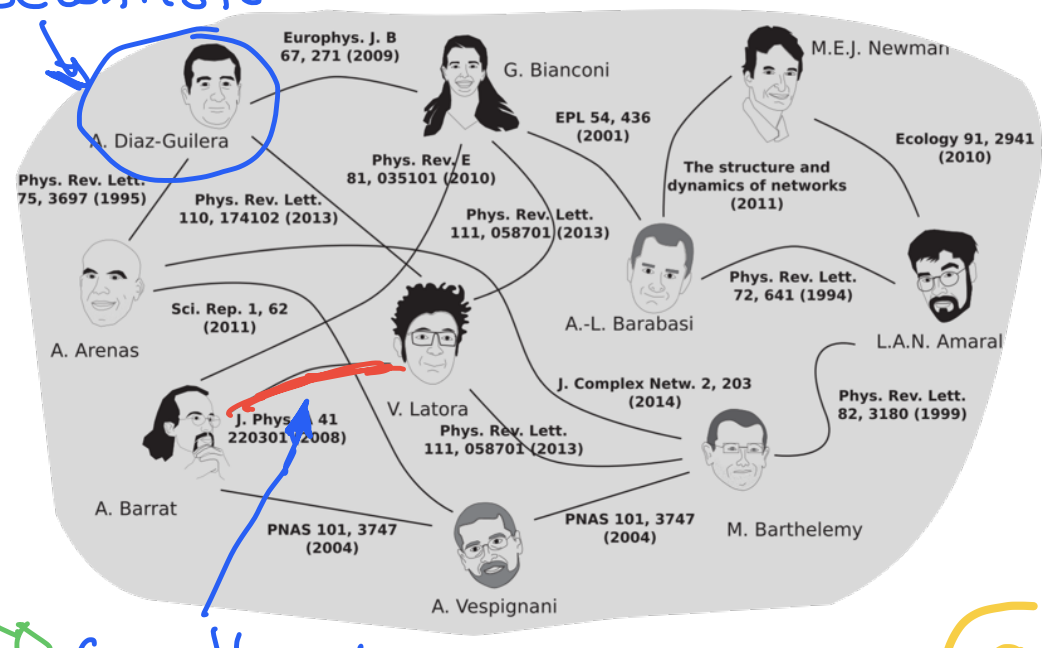
Ex Collaboration networks

MOVIE ACTORS

NODES → actors

SCIENTIFIC COLLABORATION

NODES → Scientists



LINKS → Coacting

→ coauthored papers

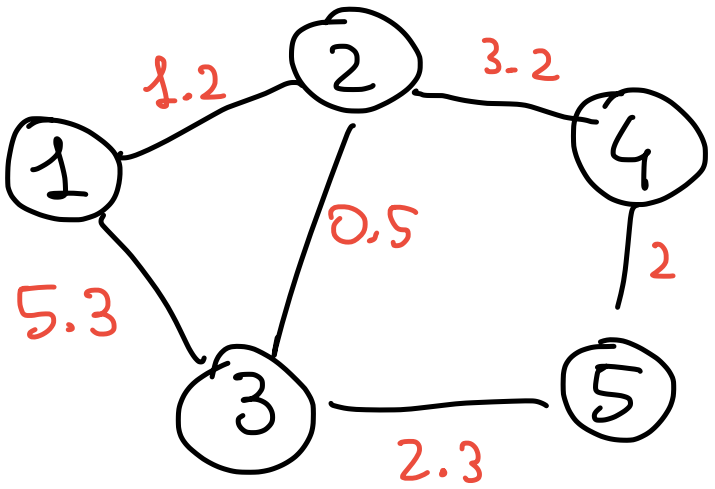
DEF WEIGHTED NETWORKS

A weighted link (i,j) is a link to which we assign an integer or real number W_{ij} indicating the intensity of the interaction.

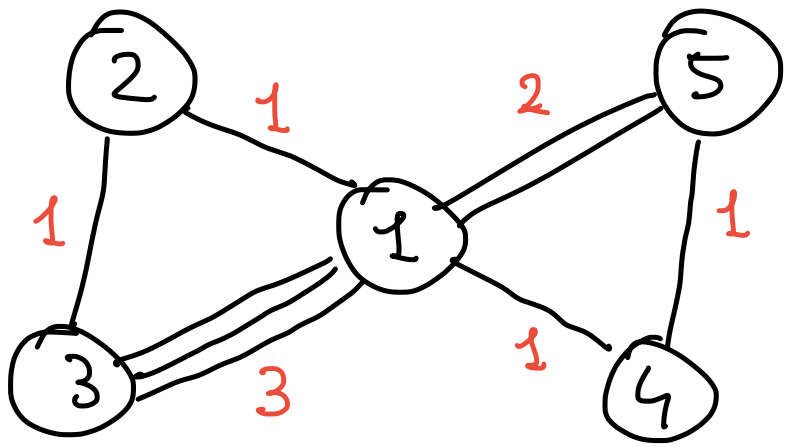
When the weight is integer the link is also called a MULTIPLE link

A weighted network is a network where all links are weighted

EX



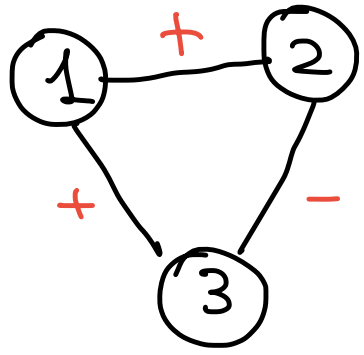
Internet



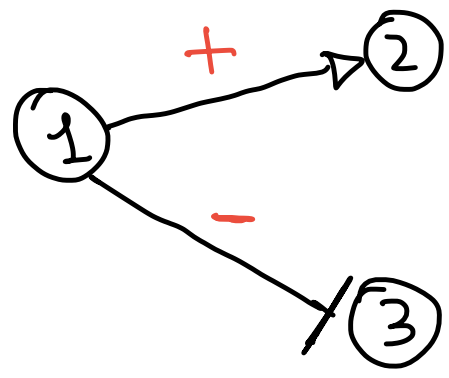
Coauthorship networks

Networks can also be "signed" when interactions of opposite type are present

EX



A social network with + indicating friends and − indicating enemies



A gene regulaton network in which a gene can either be an ACTIVATOR (+) or INHIBITOR (−) of another gene

DEF SIGNED NETWORKS

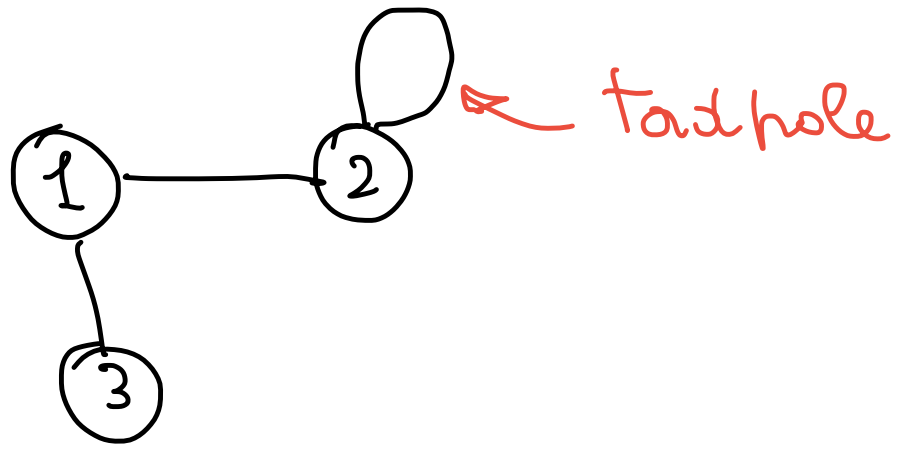
A signed link is a link to which we assign a sign (either positive or negetive). A signed network is a network where all links are signed

A special type of links are TADPOLES

DEF TADPOLE

A tadpole is a link connecting a node to itself

ES



DEF SIMPLE NETWORKS

A simple network is an undirected, unweighted, unsigned network without tadpoles