Last time: building statements using "and", "or", "not", "$\Rightarrow$".

2.7 Quantifiers

A way of turning a statement with variables into a statement that is unconditional or absolute. There are two kinds of quantifier:

universal, "for all", $\forall$, and
existential, "there exists", $\exists$.

Suppose we take a statement with a variable, e.g. $P(n) \equiv$ "n is prime". This statement is neither true nor false, but depends on n. $P(5)$ is true, but $P(4)$ is false.
However

- "For all n, n is prime", or $\forall n .\ P(n)$, is an absolute statement (which happens to be false, since $P(4)$ is false).

- "There exists n such that $P(n)$", or $\exists n.\ P(n)$, is an absolute statement (which happens to be true, since $P(3)$ is true).

There are alternative forms, e.g. "n is prime for some n".

As with summation, n is a dummy variable and can be replaced by any other name: "m is prime for some m" means the same thing.

Warning: Sometimes the universal quantification is implicit.

"If n is a positive integer, then n is prime"

"Let n be a positive integer. Then n is prime"

Can have more than one quantifier (m, n are $+^{ve}$ integers).

A. For all n there exists m such that $m > n$. (true)

B. For all n there exists m such that $n = m^2$. (false)

Order is important!

A'. There exists m s.t. for all n, $m > n$. (false)

B'. There exists m s.t. for all n, $n = m^2$ (false)

In your own time. (Challenging!) Consider $\forall n \exists m. P(n,m)$ and $\exists m \forall n\ P(n,m)$. There seem to be 4 possibilities: FF, FT, TF, TT. One of these cannot occur! Which and why?

Quiz: Write a quantified sentence for Composite$(n) \equiv$ "n is composite":

There exist a, b such that $(ab = n)$ and $(a \neq 1)$ and $(b \neq 1)$.

Interaction with negation. Examples:

For all x, $x^2 \geq x$. (false)

Negation: There exists an x s.t. $x^2 < x$. (true)

There exists n s.t. n is prime. (true)

Negation: For all n, n is composite. (false).