$P(n)$: n is prime. Then $P(3)$ is true and $P(6)$ is false.

Can have more than one variable:

$B(x,y,z)$: y lies between x and z, so $B(3,7,8)$ and $B(10,0,-10)$ are true, but $B(2,7,5)$ is false

Statements are <u>equivalent</u> if they are either both true or both false. For example if $P(n)$: $n-1=0$ and $Q(n)$: $(n-1)^2=0$ then $P(n)$ and $Q(n)$ are equivalent.

2.3 Using statements to make new statements.

As with arithmetic expressions we can combine statements but this time with "and", "or", "not" instead of $\times, +, -$ etc.

If P and Q are statements, we can consider the statement "P and Q" (also written $P \wedge Q$). Examples:

"Grass is green" and "It is Monday" (true)
"Grass is red" and "It is Monday" (false)
"Grass is red" and "It is Tuesday" (false)

Also we have the statement "P or Q" (also written $P \vee Q$).

Exs "Grass is green" or "It is Monday" True
"Grass is red" or "It is Tuesday" False
"Grass is red" or "It is Monday" True

2.4 Truth tables List all possible assignments (true or false) to the statements, and compute the outcome (true/false). We have:

P	Q	P and Q
F	F	F
F	T	F
T	F	F
T	T	T

P	Q	P or Q
F	F	F
F	T	T
T	F	T
T	T	T

Can form more complex truth tables. Eg. for

P or (Q and R)

P	Q	R	Q and R	P or (Q and R)
F	F	F	F	F
F	F	T	F	F
F	T	F	F	F
F	T	T	T	T
T	F	F	F	T
T	F	T	F	T
T	T	F	F	T
T	T	T	T	T

In your own time construct the truth table for (P or Q) and R and see if these two statements are equivalent.

As an example of the use of truth tables consider

"P and (Q or R)" and "(P and Q) or (P and R)"

× + ← analogy → × + ×

Do we have a distributive law for "and" and "or"?

Left side of room ←

P and (Q or R)

Right side of room →

(P and Q) or (P and R)

P	Q	R	P and (Q or R)		(P and Q) or (P and R)			
F	F	F	F	✓	F	F	✓	F
F	F	T	F	✓	F	F	✓	F
F	T	F	F	✓	F	F	✓	F
F	T	T	F	✓	F	F	✓	F
T	F	F	F	✓	F	F	✓	F
T	F	T	T	✓	F	T	✓	T
T	T	F	T	✓	T	T	✓	F
T	T	T	T	✓	T	T	✓	T

So P and (Q or R) is equivalent to (P and Q) or (P and R)

Also P or (Q and R) is equivalent to (P or Q) and (P or R)

2.5 Negation

not P $(\neg P)$

Examples:

P	not P
It is Monday	It is not Monday
$x \geq 3$	$x < 3$
I am happy	I am not happy.

The truth table is:

P	not P
F	T
T	F

Suppose P and Q are equivalent. Are (not P) and (not Q) equivalent? (Suppose P and Q involve some variable n, say.) Yes.

Suppose we negate some more complex statement?

P	Q = not P
$x = 2$	$x \neq 2$
$(x \leq 2)$ and $(x \geq -2)$	$(x > 2)$ or $(x < -2)$
$x = 2$ or $y = 0$	$(x \neq 2)$ and $(y \neq 0)$

Under negation it seems that "and" becomes "or" and "or" becomes "and"

<u>De Morgan's laws</u>

not(P or Q) is equivalent to (not P) and (not Q)
not(P and Q) is equivalent to (not P) or (not Q).

Check using truth tables:

P	Q	not (P or Q)	(not P)	and	(not Q)
F	F	T	T	T	T
F	T	F	T	F	F
T	F	F	F	F	T
T	T	F	F	F	F

we see these are equivalent.

2.6 Implication

This combines statements P, Q by

P implies Q / $P \Rightarrow Q$ / If P then Q ...

Informally $P \Rightarrow Q$ means "if P is true then Q is true".

If rain is forcast then I take an umbrella

If $x \geq 2$ then $x \geqslant 3$ (false)

If $x \geq 2$ then $x^2 \geqslant 3$ (true)

The only way for $P \Rightarrow Q$ to be false is if P is true and Q is false. In terms of a truth table:

P	Q	$P \Rightarrow Q$
F	F	T
F	T	T
T	F	F
T	T	T

Note that $(1=2) \Rightarrow (3>4)$!

"$\Rightarrow$" does not connote any kind of causality.

$(7 \text{ is prime}) \Rightarrow (3>2)$ is a true implication.

In the example above $x \geq 2 \Rightarrow x^2 \geq 3$, implicitly, is true for all x.

Quiz: If P is equivalent to Q, what the truth value of $(P \Rightarrow Q)$ and $(Q \Rightarrow P)$?