Warning Apply $m = 2n$ to $\displaystyle\sum_{n=1}^{10}(2n) \neq \sum_{m=2}^{20} m$

$$\begin{array}{ccc} \displaystyle\sum_{n=1}^{10}(2n) & \neq & \displaystyle\sum_{m=2}^{20} m \\ \| & & \| \\ 2+4+\cdots+20 & & 2+3+4+\cdots+20 \end{array}$$

Another example:

$$S = \sum_{n=1}^{10}\left(2^{n+1} - 2^{n-1}\right) = \sum_{n=1}^{10} 2^{n+1} - \sum_{n=1}^{10} 2^{n-1}$$

Change of variable: $m = n+1$ $\qquad$ $m = n-1$

$$S = \sum_{m=2}^{11} 2^m - \sum_{m=0}^{9} 2^m$$

$$= 2^{11} + 2^{10} + \cancel{\sum_{m=2}^{9} 2^m} - \cancel{\sum_{m=2}^{9} 2^m} - 2^1 - 2^0$$

$$= 2^{11} + 2^{10} - 2 - 1$$

1.7 Products and factorials

As for summation, but with $\prod$ replacing $\sum$. E.g. $\prod_{n=1}^{3}(2n+1) = 3 \times 5 \times 7$.

Similarly to summations we can split the range:

$$\prod_{n=1}^{10}(3n^2+2) = \prod_{n=1}^{5}(3n^2+2)\prod_{n=6}^{10}(3n^2+2).$$

Also we may factorise:

$$\prod_{n=1}^{10}(x^2-1) = \prod_{n=1}^{10}(x+1)(x-1) = \prod_{n=1}^{10}(x+1)\prod_{n=1}^{10}(x-1)$$

An important product is the factorial:

$$n! = \prod_{m=1}^{n} n = 1 \times 2 \times \cdots \times n.$$

Note that $0! = 1$.

Why? We would like it to be the case that $(n+1)! = (n+1)\,n!$ (e.g. $n=2$ $3! = 3 \times 2!$).
Now set $n=0$, to get $1! = 1 \times 0!$. Since $1! = 1$ it must be the case that $0! = 1$.

Task: Write $\prod_{m=1}^{n} 2m$ in terms of simple functions (inc. factorials. Repeat with $\prod_{m=0}^{n} (2m+1)$.

$$\prod_{m=1}^{n} 2m = \prod_{m=1}^{n} 2 \times \prod_{m=1}^{n} m = 2^n n!$$

<u>2.1 Statements</u> Statements/assertions are whole sentences. They are either true or false. (Sometimes we don't know which.)

Examples

"It is sunny"
"Berlin is the capital of Germany"
"Aberystwyth is the capital of France"
"17 is prime"
"$16 > 17$"
"$P = NP$"

} statements

"The capital of Bangladesh"
"$x+1$"
"the set of prime numbers"

} non-statements

We like to give statements labels:

P is the statement: "3 is prime".

2.2 Statements with variables.

"$x = 4$", "n is odd", "there is a prime number larger than n", "$A \subseteq B$".

(x is a number, n is a whole number, A, B are sets).