<u>Recap</u> • We looked at ways to count sets of objects, including
- pairs, where $|A \times B| = |A| \times |B|$, and
- permutations of n objects; there are $n!$ of these.

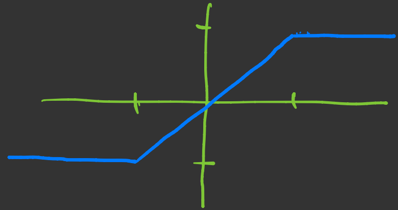
• The power set of a set X is the set of all subsets of X:
$\mathcal{P}(X) = \{A : A \subseteq X\}$. If X is finite, $|\mathcal{P}(X)| = 2^{|X|}$.

• Let X be a finite set with $|X| = n$. The number of subsets of X of size k is denoted $\binom{n}{k}$. We proved that $\binom{n}{k} = \dfrac{n!}{k!(n-k)!}$

• We defined function from A to B, written $f : A \to B$. A is the domain and B the codomain. For each $a \in A$, the function f specifies a corresponding $b = f(a)$ in B. We say that f maps a to b.

<u>Examples</u>
• $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$, for all $x \in \mathbb{R}$. } different
• $f : \mathbb{N} \to \mathbb{N}$ defined by $f(n) = n^2$. } functions!
• $f : \mathbb{N} \to \mathbb{Z}$ defined by $f(n) = n-1$. (NB $\mathbb{Z}$, not $\mathbb{N}$).
• $f : \mathbb{R} \to \mathbb{R}$

$$f(x) = \begin{cases} -1, & \text{if } x \le -1 \\ x, & \text{if } -1 < x < 1 \\ 1, & \text{if } x \ge 1 \end{cases}$$



Definition by cases.

- $f : \mathbb{N} \to \{F, T\}$

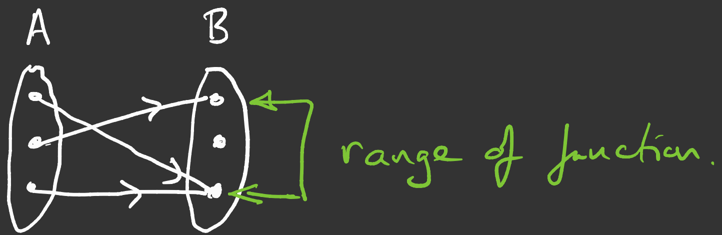
$$f(n) = \begin{cases} T, & \text{if } n \text{ is even;} \\ F, & \text{otherwise.} \end{cases}$$

- X is a finite set. $f : \mathcal{P}(X) \to \mathbb{N} \cup \{0\}$ given by $f(A) = |A|$.
- $f : \mathcal{P}(X) \to \mathcal{P}(X)$, given by $f(A) = X \setminus A$.

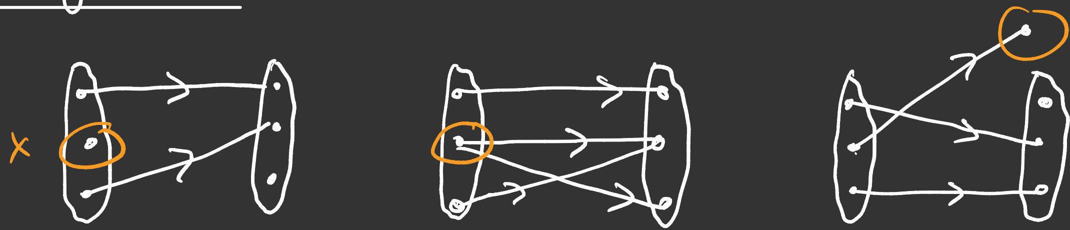
Non-examples

- $f : \mathbb{N} \to \mathbb{N}$ given by $f(n) = n - 1$. ($f(1) = 0$ is not in the codomain)
- $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = 1/x$. (Not defined at $x = 0$.)
- $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = \begin{cases} 0, & \text{if } x \le 0; \\ 1 & \text{if } x \ge 0. \end{cases}$ (Multiply defined at 0.)

Picturing functions



... and non-functions



Definition Suppose $f : A \to B$ is a function. The range of f is $\{f(a) : a \in A\}$.

Codomain : all values that $f(a)$ can possibly take.

Range : all values of $f(a)$ that actually occur.

Examples

- $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$. $\text{Range}(f) = \{x \in \mathbb{R} : x \geq 0\}$
  $= [0, \infty)$.
- $f : \mathbb{N} \to \mathbb{N}$ defined by $f(n) = 2n$. $\text{Range}(f) = \{n \in \mathbb{N} : n \text{ is even}\}$.

- $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = \begin{cases} x, & \text{if } x \text{ is rational;} \\ 0, & \text{if } x \text{ is irrational.} \end{cases}$

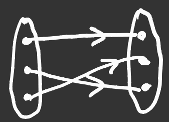
$\text{Range}(f) = \mathbb{Q}$

6.2 Injective, surjective, bijective functions

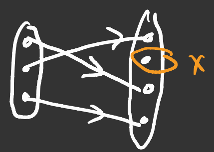
Definition. Suppose $f : A \to B$ is a function.

- f is injective if $f(a_1) \neq f(a_2)$ whenever $a_1, a_2 \in A$ and $a_1 \neq a_2$.
- f is surjective if for all $b \in B$ there exists $a \in A$ such that $f(a) = b$.
- f is bijective if it is injective and surjective.

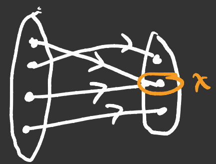
In pictures



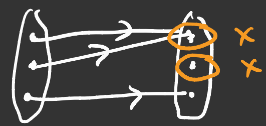
I & S



I & not S



not I & S



not I & not S.

Examples $f : \mathbb{Z} \to \mathbb{Z}$, defined by $f(n) = 3 - n$. Injective ✓ Surjective ✓

(Suppose $f(a_1) = f(a_2)$. Then $3 - a_1 = 3 - a_2 \Rightarrow a_1 = a_2$, so injective.
Suppose $b \in \mathbb{Z}$. Then $f(3-b) = 3 - (3-b) = b$, so surjective.)