## 1.1 Basic arithmetic operations $+ \quad - \quad \times \quad \div$

Usually $\times \to$ juxtaposition : $3 \times x \to 3x$, but <u>not</u> $x \times 3 \to x3$, and certainly not $2 \times 3 \to 23$.

Usually $\div$ is written $x/y$, $\frac{x}{y}$ not $x \div y$.

## 1.2 Brackets

Used to control the order of operations. Sometimes we can omit the brackets (BIDMAS). Note that addition and subtraction have the same precedence. E.g. $4 - 2 + 3$ is 5 and not $4 - (2+3) = -1$ (if + binds more tightly).

Sometimes we <u>do</u> need the brackets : $(w + x) \times (y + z)$ which by BIDMAS would be $w + (x \times y) + z$.

Some operators are <u>associative</u> : we don't need to bracket sequences of these. E.g. $(2+3)+4 = 2+(3+4)$, but $(8/4)/2 \neq 8/(4/2)$.

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Question. Define $\circ$ by $x \circ y = xy + x + y$. So $2 \circ 3 = 2 \times 3 + 2 + 3 = 11$. Is $\circ$ associative? I.e. is it the case that $(x \circ y) \circ z = x \circ (y \circ z)$ for all $x, y, z$?

Yes!

$$(x \circ y) \circ z = (xy + x + y) \circ z = xyz + xz + yz + xy + x + y + z$$

$$x \circ (y \circ z) = x \circ (yz + y + z) = xyz + xy + xz + x + yz + y + z$$

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<u>1.4 Greek letters</u> Mathematicians use them. l.c., only a few u.c. $\Delta$, $\Gamma$.

## 1.3 Infinity

$\infty$ is not a number. But we do use it in situations: $\sum_{n=1}^{\infty} x_n$ or $\int_0^\infty \text{—}\, dx$

## 1.5 $\Sigma$-notation

"sigma"

Used to denote summation, e.g.,

$$\sum_{n=1}^{10} n = 1+2+3+\cdots+10 = 55.$$

General form is

$$\sum_{n=a}^{b} x_n$$

range of summation

summand

dummy variable.

$\sum_{n=a}^{b} x_n = \{\text{something involving } n\}$ is wrong!

Meaning: take the values $x_a, x_{a+1}, \ldots, x_b$ and add them together.

Exs

$$\sum_{n=1}^{4} n^2 = 1+4+9+16 = 30$$

$$\sum_{n=-10}^{10} n^3 = (-10)^3 + (-9)^3 + \cdots + 0 + \cdots + 9^3 + 10^3$$

$$= 0 .$$

We can replace the dummy variable by some other without change

$$\sum_{n=a}^{b} n^2 = \sum_{m=a}^{b} n^2 .$$

Alternative notation:

$$\sum_{n=a}^{b} x_n = \sum_{a \le n \le b} x_n = \sum_{n \in [a,b]} x_n .$$

Limits can be expressions:

$$\sum_{n=1}^{m} n = \frac{m(m+1)}{2}$$

Upper limit can be $\infty$:

$$\sum_{n=1}^{\infty} 2^{-n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = 1.$$

Range may be an arbitrary set of numbers:

$$\sum_{p \in \mathbb{P}} \frac{1}{p}$$ where $\mathbb{P}$ is the set of all prime numbers.

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{11} + \dots$$

1.6 Manipulating sums.

Split the summand: $\sum_{n=1}^{\infty} \left( \frac{1}{n^2} + \frac{1}{n^3} \right) = \sum_{n=1}^{\infty} \frac{1}{n^2} + \sum_{n=1}^{\infty} \frac{1}{n^3}$

Split up the range:

$$\sum_{n=1}^{\infty} \frac{n}{2^n} = \underbrace{\sum_{n=1}^{10} \frac{n}{2^n}}_{\text{calculate}} + \underbrace{\sum_{n=11}^{\infty} \frac{n}{2^n}}_{\text{estimate.}}$$

Substitute for dummy variable:

$$\sum_{n=1}^{20} (n-1)^3 \qquad \text{Substitution } n-1 \to m$$

We get $\displaystyle\sum_{m=0}^{19} m^3$

Example of use of substitution:

$$S = \sum_{n=1}^{100} \left(n^3 - (n-1)^3\right)$$

$$= \sum_{n=1}^{100} n^3 - \sum_{n=1}^{100} (n-1)^3 \qquad \text{Substitute } m = n-1$$

$$= \sum_{m=1}^{100} m^3 - \sum_{m=0}^{99} m^3$$

$$= 100^3 + \cancel{\sum_{m=1}^{99} m^3} - \cancel{\sum_{m=1}^{99} m^3} - 0^3$$

$$= 10^6 .$$