Q3 (b)

Let $A = 1 - 4x^2 - y^2$

$$\dot{x} = xA - \tfrac{1}{2}y(1+x), \quad \dot{y} = yA + 2x(1+x)$$

At $(0,0)$: $\boxed{\begin{aligned}\dot{x} &= x - \tfrac{1}{2}y \\ \dot{y} &= y + 2x\end{aligned}}$ is the linearised system

$$J = \begin{bmatrix} 1 & -\frac{1}{2} \\ 2 & 1 \end{bmatrix} \qquad (\lambda - 1)^2 + 1 = 0$$

$$\lambda = 1 \pm i$$

unstable spiral at the origin.

note Jacobian at $(0,0)$ is easy to spot, as the system is a Taylor Series

$V = A^2$

$$\frac{dV}{dt} = 2A(-8x)\left(xA - \tfrac{1}{2}y(1+x)\right) + 2A(-2y)\left(yA + 2x(1+x)\right)$$

$$= 2A\left[-8x^2A + 4xy(1+x) - 2y^2A - 4xy(1+x)\right]$$

$$= 2A(-2)\left(+(4x^2 + y^2)\right).$$

$$= -4A\left(4x^2 + y^2\right)$$

$\therefore \frac{dV}{dt} < 0$ for $4x^2 + y^2 \neq 1$ and $4x^2 + y^2 + 1 \Leftrightarrow (x,y) = 0$

V decreases in the region $\mathbb{R}^2 \setminus \{E \cup \{0\}\}$.

$\therefore V \to 0 \Rightarrow V(x(t), y(t)) \to 0$ i.e.

$(x(t), y(t)) \to E$