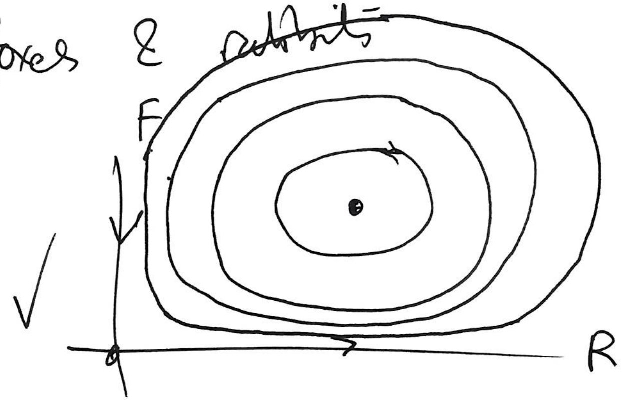
T10.1

foxes & rabbits



$x = 1, \ y = 1$

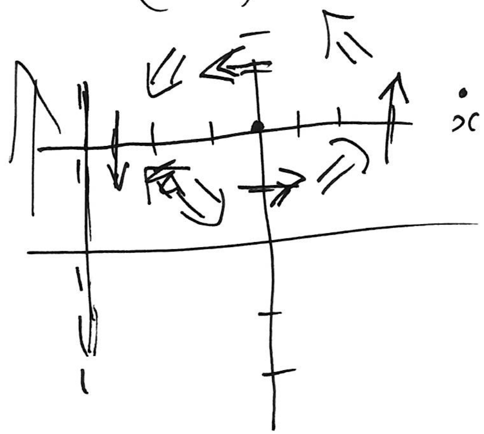
Competing species

$\dot{x} = x(1 - y)$ $\qquad$ $y > 1$

$\dot{y} = -y(1 - x)$

$$\begin{bmatrix} 1 & -x \\ y & -1 \end{bmatrix}\Bigg|_{(1,1)} = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$$

Det $\quad (\lambda - 1)(\lambda + 1) + 1 = \lambda^2 + 1 = 0 \quad \underline{\lambda = \pm i}$



$\dot{x} = 0$

$\dot{x} = 0 \qquad x = 0, \ y = 1$

$\dot{y} = 0 \qquad y = 0 \quad x = 1$

TID 2

$$\frac{dy}{dx} = \frac{-y(1-x)}{x(1-y)}$$

$$0 = \int \frac{1-x}{x}\,dx + \int \frac{1-y}{y}\,dy$$

$$C = \ln(x) - x + \ln(y) - y$$

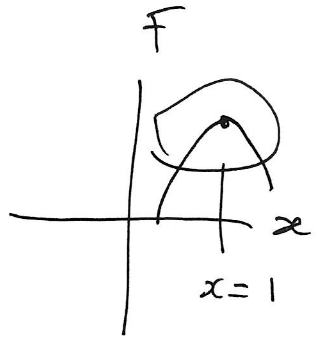
$$C = \ln(xy) - x - y$$

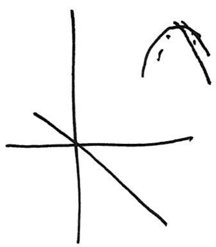
$$C' = xy\,e^{-x}e^{-y}$$

$$= xy\,e^{-(x+y)}$$

$$V(x,y) = xe^{-x} \cdot ye^{-y}$$

$$= F(x) \cdot F(y)$$





$$F(x) = xe^{-x}$$

$$F'(x) = e^{-x} - xe^{-x}$$

$$= e^{-x}(1-x)$$

$$F(1) = 1$$

max in $F(x)$ & $F(y)$
at $(x,y) = (1,1)$

$F'(x) = 0$ $\quad x = 1$

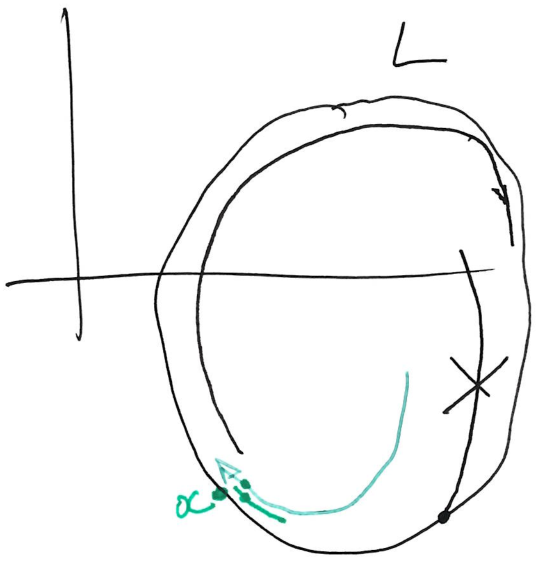
$F'(y) = 0$ $\quad y = 1$

$$F''(x) = -e^{-x} - e^{-x} + xe^{-x}$$

$$F''(1) = -2e^{-1} + e^{-1} = -e^{-1} < 0$$

∴ closed contours at a maximum.

Limit cycle



uniqueness of trajectories
nearby trajectories do not "hit" the limit cycle, they spiral closer & closer.

$t_n \qquad x(t_n) \to x$

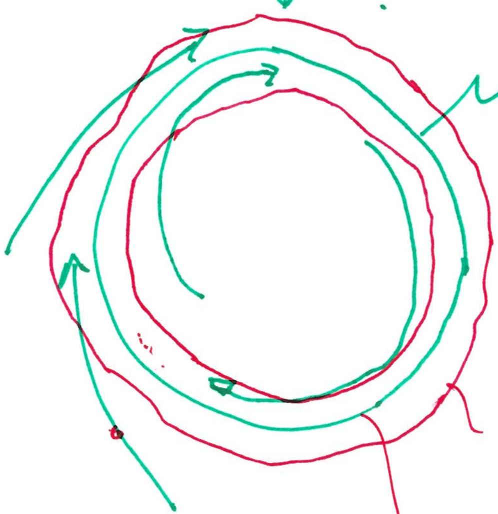
$x(t), \quad x(0) = x_0$

x_0

$x(t) \to x^*$ as $t \to \infty$.

T10.4

Limit cycle



closed orbit

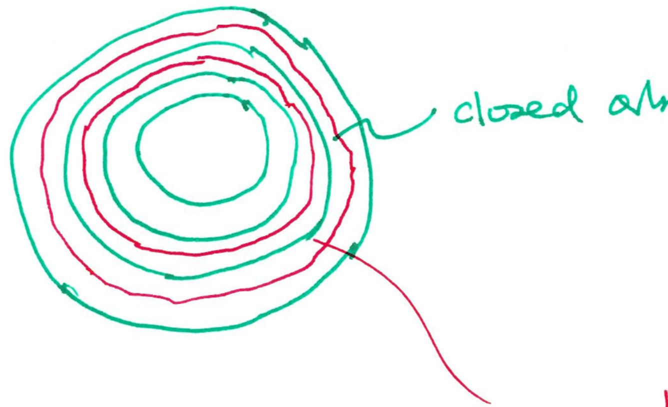
tubular nbd.

nbd

no other closed orbit.

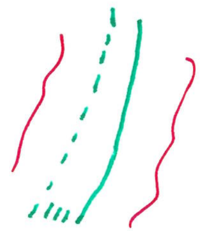
just one

asymptotically stable



closed orbit

stable.



infinity of closed orbits in every open nbd.

AS

stable.