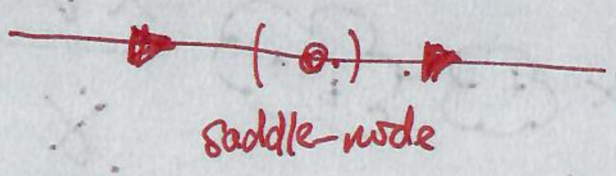
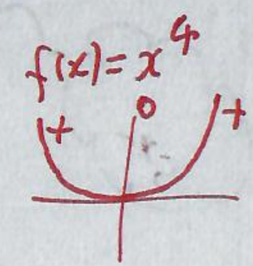
The fixed point at $x = 0$ of the ODE $\dot{x} = x^4$ is

Select one:

- (•) unstable ✓
- ( ) both stable and unstable
- ( ) asymptotically stable
- ( ) stable
- ( ) none of these

Clear my choice





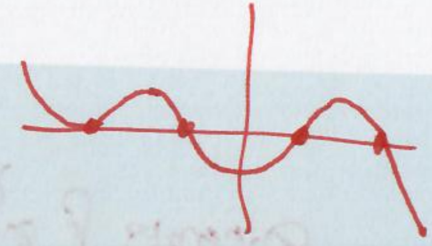
unstable is complement of stable.
negative

asymptotically stable requires stability and then approach to fixed pt as $t \to \infty$ within the zone of stability
nbd

Which of the following ODEs on $\mathbb{R}$ give a phase portrait which is qualitatively the same as the one illustrated below?

+   +   −   +   −

$\frac{dx}{dt} > 0$ R , $\frac{dx}{dt} < 0$ L



Select one or more:

- [x] $\dot{x} = +x^4(x-1)(x-2)(3-x)$ ~ $-x^7$ deg 7? ✓✓   even deg poly $|x| \to \infty$. $+ +$ / $- -$
- [ ] $\dot{x} = x(x-1)(x-2)(x-3)$ deg 4 ✗ $+ +$
- [x] $\dot{x} = -x^2(x-2)(x-3)(x-4)$ deg 5 ✓✓   odd deg poly. $|x| \to \infty$ $+ -$ / $- +$
- [ ] None of these.   ✗
- [ ] $\dot{x} = x^4(x-2)(x-3)(x-4)$ deg 7 ✓? ✗ $+x^7$ $- +$ ✗   ~ $-x^5$

$\dot{x} = -x^2(x-1)(x-2)(x-3)$ deg 5 ✓

The system

$$\dot{x} = rx + x\frac{(1-2x^2)}{(1-3x^2)}$$

has a bifurcation at the point (?) , and of type (?):

- [ ] $(x, r) = (0, -1)$ and supercritical pitchfork
- [ ] None of these
- [ ] $(x, r) = (0, 1)$ and supercritical pitchfork
- [x] $(x, r) = (0, -1)$ and subcritical pitchfork
- [ ] $(x, r) = (0, 1)$ and subcritical pitchfork
- [ ] $(x, r) = (0, 0)$ and supercritical pitchfork

$\dot{x} = rx + x\frac{(1-2x^2)}{(1-3x^2)}$

$x = 0$? gives a fixed point of system

$\dot{x} = rx + x(1-2x^2)(1-3x^2)^{-1}$

$= rx + x(1-2x^2)\left(1+3x^2+\frac{(3x^2)^2}{2!}+\dots\right)$

$= rx + x(1-2x^2+3x^2+H.O.T)$

$= rx + x(1+x^2) + H.O.T$

$= (r+1)x + x^3 + H.O.T$

$= \mu x + x^3 + H.O.T.$

Normal form for a Pitchfork.

$= x(\mu + x^2)$

Subcritical for μ. $\mu = r+1$

Subcritical Pitchfork for r.

3 solns $\mu < 0$ $x = \pm\sqrt{-\mu}$

1 soln $\mu = 0$ $x = 0$

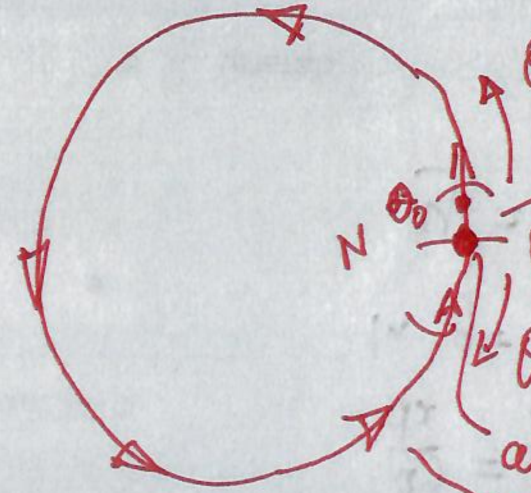
1 soln $\mu > 0$ $x = 0$.

Real

Consider the ODE on the circle $\mathbb{S}$ given by $\dot{\theta} = 1 - \cos(\theta)$. The unique fixed point of the ODE is:

Select one or more: not?

- [ ] a. asymptotically stable (not stable)
- [ ] b. None of these. ×
- [ ] c. neither stable nor unstable ×
- [x] d. attracting
- [x] e. unstable
- [ ] f. stable ×



θ incr (anticlockwise)

saddle node unstable

$\theta = 0\ (2\pi)$

θ decr (clockwise)

all points θ_0 have orbits (soln curve) which approach $\theta = 0$ as $t \to$ increases ∞.

fixed pts $\dot{\theta} = 0$

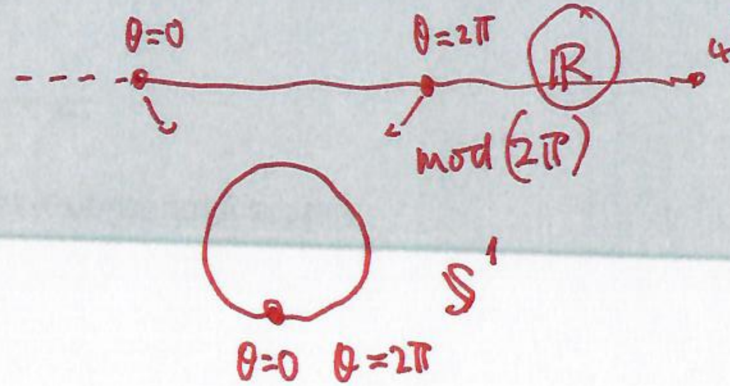
$\dot{\theta} = g(\theta) = 1 - \cos\theta$

$g(\theta) = 1 - \cos\theta = 0 \Rightarrow \cos\theta = 1$

$g(\theta) > 0$

$\theta = \frac{\pi}{2}$ is the only on the circle

$\theta = \frac{5\pi}{2} \left(= \frac{\pi}{2}\right)$



Remind yourself of the definitions of

- unstable fixed point
- stable fixed point, but not asymptotically stable
- asymptotically stable fixed point
- attracting fixed point

Which is the type of fixed point at the origin for each of the following systems on $\mathbb{R}^2$

$\dot{x} = y, \dot{y} = -4x$ - stable fixed point, but not asymptotically stable

$\dot{x} = -2x, \dot{y} = -3y$ - asymptotically stable fixed point

$\dot{x} = 0, \dot{y} = 0$ - stable fixed point, but not asymptotically stable

$\dot{x} = 0, \dot{y} = y$ - unstable

and on the circle $\mathbb{S}$ at the fixed point $\theta = 0$ for each of

$\dot{\theta} = 1 - \cos(\theta)$ - unstable | unstab

$\dot{\theta} = \sin^2(2\theta)$ - unstable | none of tl

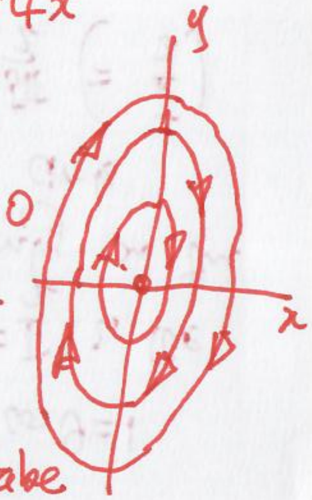
$\dot{x} = \cancel{y}\ \dot{y} = -x$

$\dot{x} = y\ ,\ \dot{y} = -4x$

$\frac{dy}{dx} = \dot{y}/\dot{x} = -\frac{4x}{y}$

$\int y\,dy + \int 4x\,dx = 0$

$\frac{y^2}{2} + 2x^2 = \text{const.}$



Stable

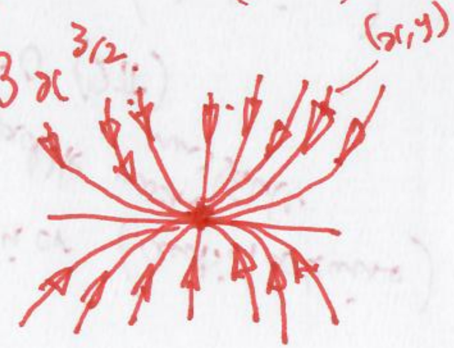
not asymptotically stable

$\dot{x} = -2x\ ,\ \dot{y} = -3y$

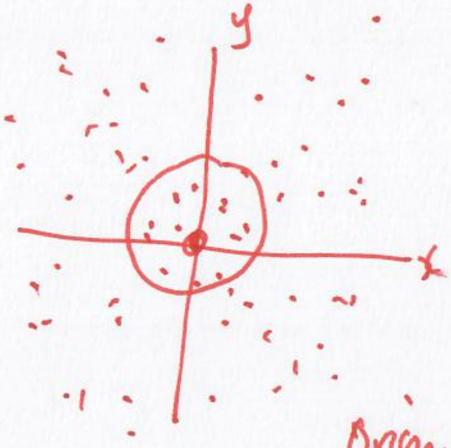
$\frac{dy}{dx} = \frac{3y}{2x} \Rightarrow \int \frac{dy}{y} = \int \frac{3}{2}\frac{dx}{x}$

$\ln y = \frac{3}{2}\ln x + C (= \ln B)$

$y = Bx^{3/2}$



$\dot{x}=0,\ \dot{y}=0$

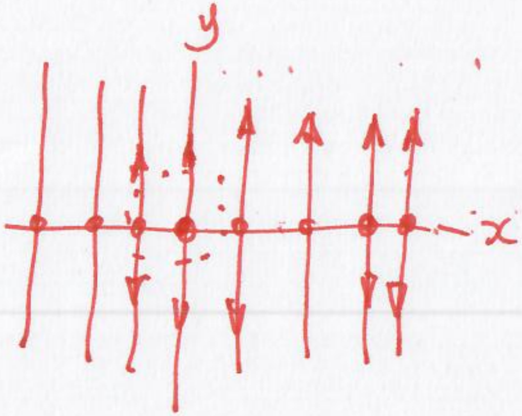


plane of fixed pts.

every point is a fixed pt.

Origin stable but $\underline{\underline{\text{not}}}$ asymptotically stable

$\dot{x}=0 \quad \dot{y}=y$



$\dot{x}=0 \Rightarrow x=x_0$, const, $x_0 \in \mathbb{R}$

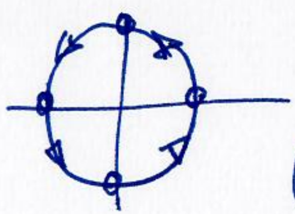
in any nbd of 0 $\exists$ orbits escaping to infinity

unstable

$\dot{x}=0,\ \dot{y}=0$

$\forall$ $y=0$

$\dot{\theta}=1-\cos\theta$, unstable fixed pt at $\theta=0$



unstable
(saddle-node)