

Tutorial 4 "x=0" !!!??

4.1

Reference z.b p14.

$$\dot{x} = r \ln(x) + x - 1, \quad x > 0$$

NOTICED $x=1$, equilibrium/fixe point.

$$x = 1+y \quad (\text{local coordinates})$$

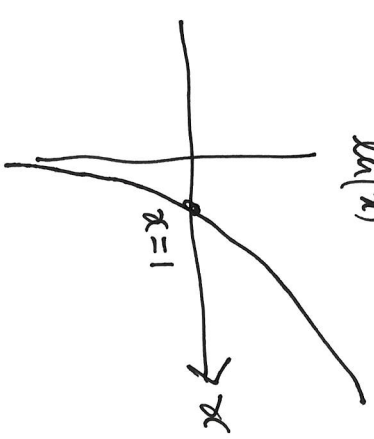
$$\dot{y} = r \ln(1+y) + 1+y-1$$

$$\dot{y} = r \ln(1+y) + y.$$

↑
power series

Eqn is not defined (fn value no sense at $x=0$)

$$y = -1? \\ \ln(x)$$



Consider only excluded $x > 0$ ✓ as ~~the~~ $\ln(x)$ is defined only for $x > 0$.

$$\dot{x} = r + \frac{1}{2}x - \frac{x}{1+x}$$

$x = -1$ gives singular value.

4.2

exclude $x = -1$, but we can consider $x < -1$, $x > -1$

Two regions to consider for phase:

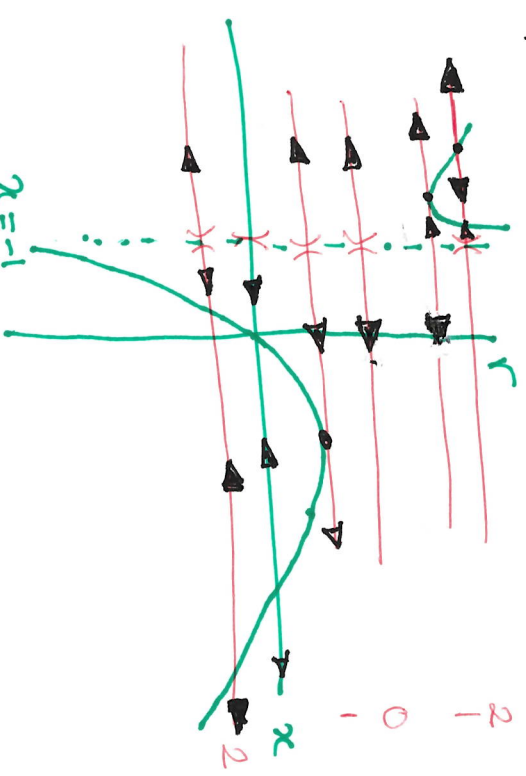
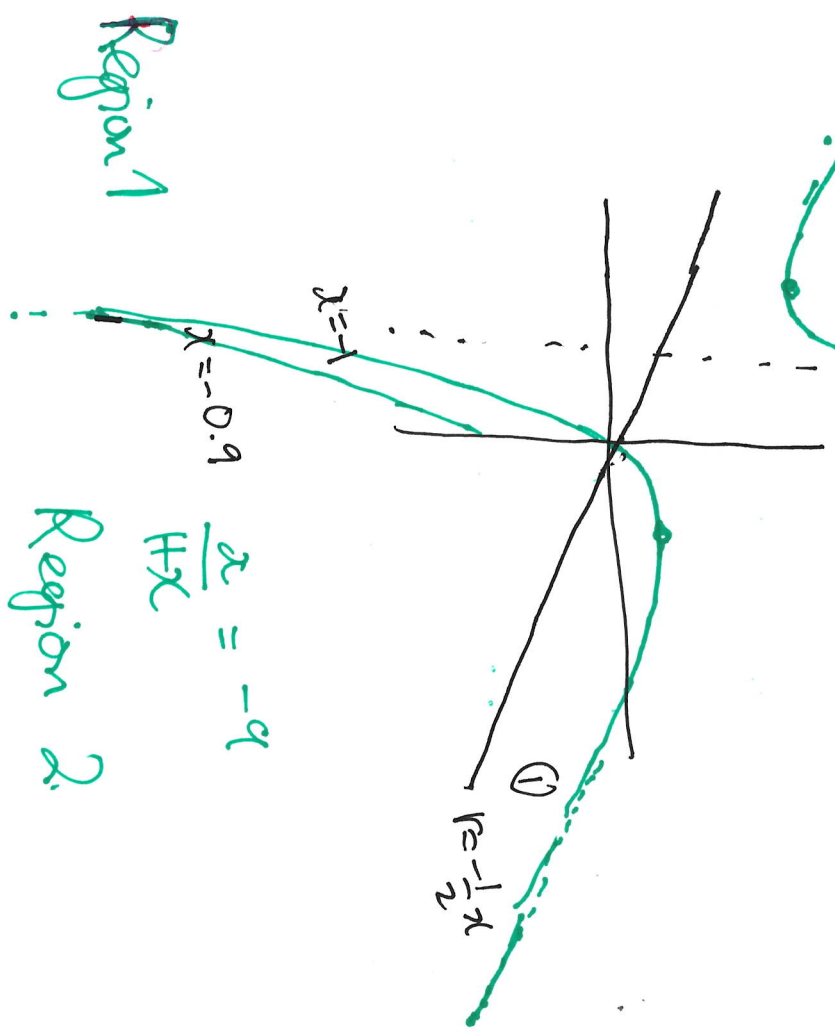
FP set $v = -\frac{1}{2}x + \frac{x}{1+x}$

Max and min
1st quad. 2nd quad.

Bifurcation diagram.

$$f(x) = r + \frac{1}{2}x - \frac{x}{1+x} \rightarrow \infty \text{ as } x \rightarrow \infty$$

$$f(x) = r + \frac{1}{2}x - \frac{x}{1+x} \rightarrow -\infty \text{ as } x \rightarrow -\infty$$



FPs
2
1
0
1

Look like 2 saddle nodes (x_1^*, r_1^*) , (x_2^*, r_2^*) are the hypercritical point?

\checkmark min max

introduce

$$\begin{cases} y_1 = x - x_1^* \\ \mu_1 = r - r_1^* \end{cases}$$

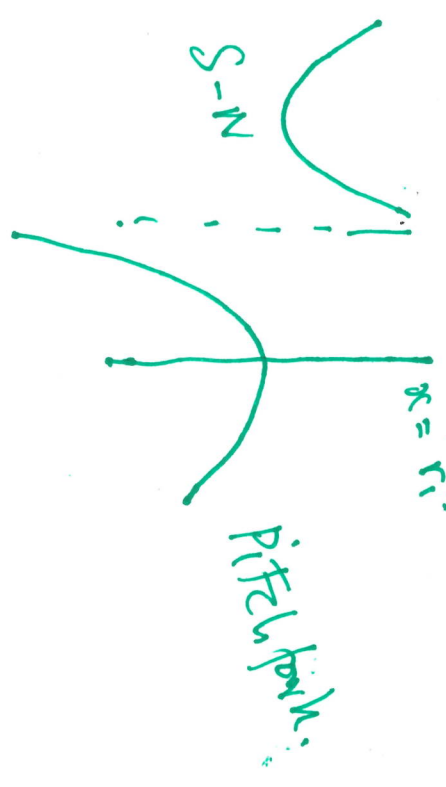
→ SN normal form in y_1, μ_1 coordinates

$$\begin{cases} y_2 = x - x_2^* \\ \mu_2 = r - r_2^* \end{cases}$$

→ SN normal form in y_2, μ_2 coordinates

$$\dot{x} = (x - r_1^*)$$

Note



To check a bifurcation diagram of $\dot{x} = f(x, r)$ (4.4)

Sketch plot $\dot{x} = f(x, r)$, $\dot{r} = 0$
dynamical flow in the $x-r$ plane

$$\dot{x} = x^3 - rx$$

(trivial but ok) + r is a parameter
i.e. constant AS

Dynamically, I can introduce r with the
condition $\dot{r} = 0$
vector field on \mathbb{R}^2

$$(\dot{x}, \dot{r}) = (x^3 - rx, 0)$$

Maths Applet - has a sketch of it

To get bifurcation diagram - ① plot FP diagram

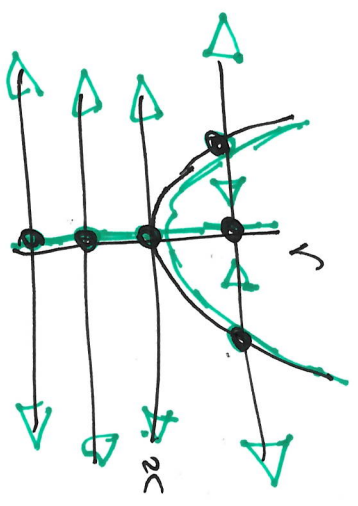
② add directional indicators of $\frac{dx}{dt} > 0, \frac{dx}{dt} < 0$ or $\frac{dx}{dt} = 0!$ in increasing time

Leading Term

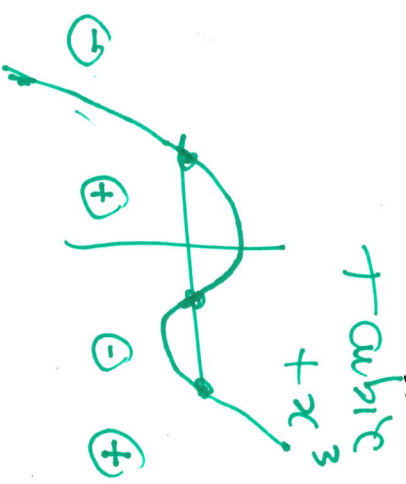
$$\dot{x} = x^3 - rx$$

$$= x(x^2 - r)$$

x^2	+	+
x^3	-	+
$-x^2$	-	-



Leading bifurcation



3.1.4 ~~reference~~ in Exercise 2)