TUTORIAL 3 -

Quiz 1 Find the fixed point set of $\dot{x} = x^5 - 2\alpha x^3 + x$

FP set : $x(x^4 - 2\alpha x^2 + 1) = 0$.

$$\begin{cases} x \equiv 0 & \text{for all } \alpha \\ x^2 = \alpha \pm \sqrt{\alpha^2 - 1} \end{cases}$$

we require real soln's $\Rightarrow$ (1) $\alpha^2 - 1 \geq 0$ and (2) $x^2 \geq 0$

(1) $\alpha^2 - 1 \geq 0 \Rightarrow \alpha \geq 1$ or $\alpha \leq -1$

For $\alpha \geq 1$, $\alpha + \sqrt{\alpha^2 - 1} > 0$ and $\alpha - \sqrt{\alpha^2 - 1} > 0$ since $\sqrt{\alpha^2 - 1} < \sqrt{\alpha^2} = \alpha$.

For $\alpha \leq -1$ $\alpha + \sqrt{\alpha^2 - 1} < 0$ and $+\alpha - \sqrt{\alpha^2 - 1} < 0$

So we have real solns for $\alpha \geq 1$

$$x^2 = \alpha + \sqrt{\alpha^2 - 1} \Rightarrow x = \pm\sqrt{\alpha + \sqrt{\alpha^2 - 1}}$$

$$x^2 = \alpha - \sqrt{\alpha^2 - 1} \qquad x = \pm\sqrt{\alpha - \sqrt{\alpha^2 - 1}}$$

$$x = 0 \quad \forall \alpha \qquad x = 0.$$

Observations $x = \pm\sqrt{\alpha + \sqrt{\alpha^2 - 1}}$: $x \to \pm\infty$ as $\alpha \to \infty$

$x \to \pm 1$ as $\alpha \to 1$

$x = \pm\sqrt{\alpha - \sqrt{\alpha^2 - 1}}$: $x \to \pm 0$ as $\alpha \to \infty$

$x \to \pm 1$ as $\alpha \to 1$

1.2

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Plot[(x^4 + 1) / (2 x^2), {x, -3, 3}]
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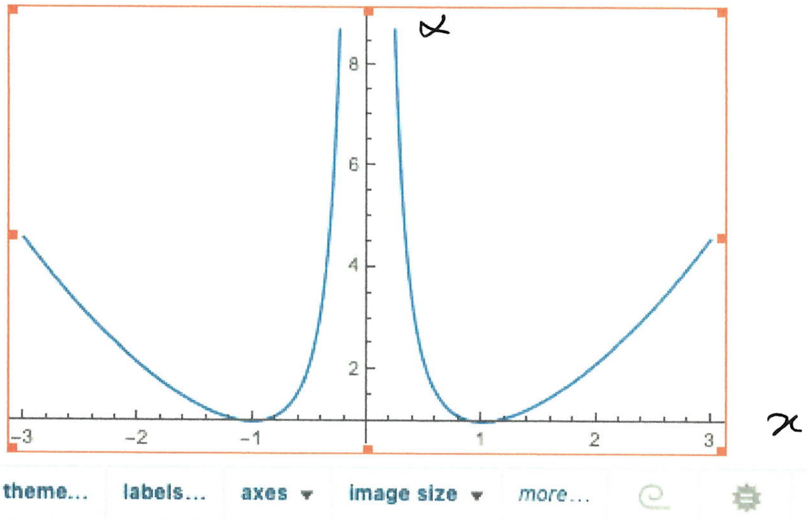
Out[4]=



Note: this satisfies the asymptotics on the previous page.

Obtained plot by interpreting $x^4 + 2\alpha x^2 + 1 = 0$

as $\alpha = (x^4+1)/(2x^2)$

[Note also it is what we suggested originally on the board until we got lost in too much detail]

So we have

$\alpha > 1$ 5 solutions (fixed points)

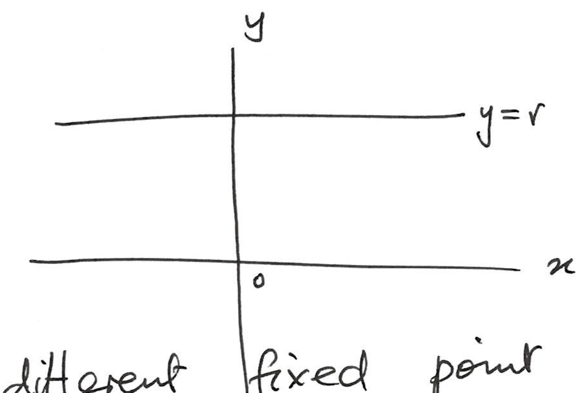
$\alpha = 1$ 3 solutions (fixed points)

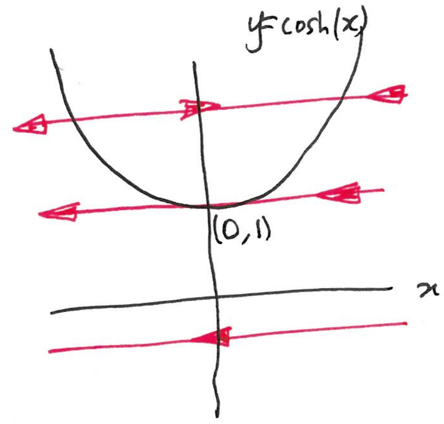
$\alpha < 1$ 1 solution (fixed points)

3.1.2

$$\dot{x} = r - \cosh(x)$$

FP set $\quad y = r \quad \& \quad y = \cosh(x)$





We see three ranges of r with different fixed point structures

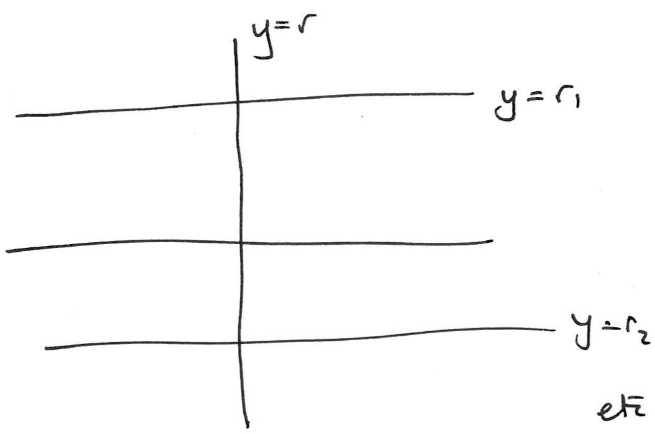
$r > 1$, 2 fixed points

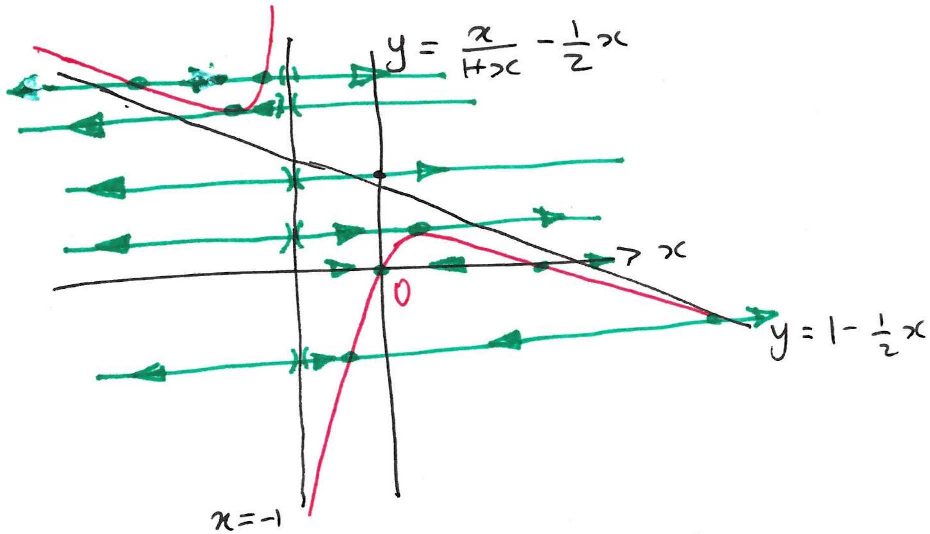
$r = 1$, 1 fixed point

$r < 1$, 0 fixed point

3.1.4 I also briefly mentioned

$\dot{x} = r + \frac{1}{2}x - \frac{x}{1+x}$. For fixed points, graph $y = r$ & $y = \frac{x}{1+x} - \frac{1}{2}x$ and see where they intersect: because then $r = \frac{x}{1+x} - \frac{1}{2}x$ which is the eq^n for finding fixed point





$y = r$ is an horizontal line chosen according to the value of r we are interested in.

$y = \frac{x}{1+x} - \frac{1}{2}x$ has an asymptote at $x = -1$

$x \to -1^-$ & $y \to \infty$

$x \to -1^+$ & $y \to -\infty$

As $x \to \infty$ $\frac{x}{1+x} \to 1$ &

$y \simeq 1 - \frac{1}{2}x$ as $x \to \pm\infty$.

Need to investigate maximum pt M
minimum pt m

Expect saddle node bifurcations