Tutorial Class 1

T.1.1

$\dot{x} = f(x)$, differentiable f

Dynamics of x, needed the regions of $+, -, =0$ of f

$gr(f)$ was vital.

2.2.1 (Strogatz p37)

$\boxed{\dot{x} = 4x^2 - 16.} = 4(x-2)(x+2)$

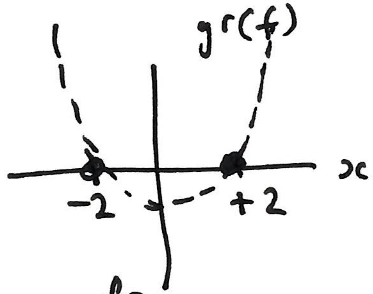
$\downarrow$

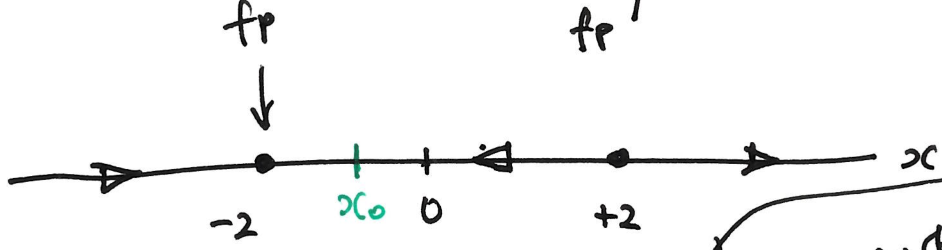
~~zeroes~~ of f

= fixed pts of

$\dot{x} = f(x)$

$f(x_0) = 0 \iff x(t) = x_0$ is a soln "fixed pt"





phase portrait of $\dot{x} = f(x)$

$x(t)$, $x(0) = x_0$

$t \to \infty$, $x(t) \to -2$

$t \to -\infty$ $x(t) \to 0$.

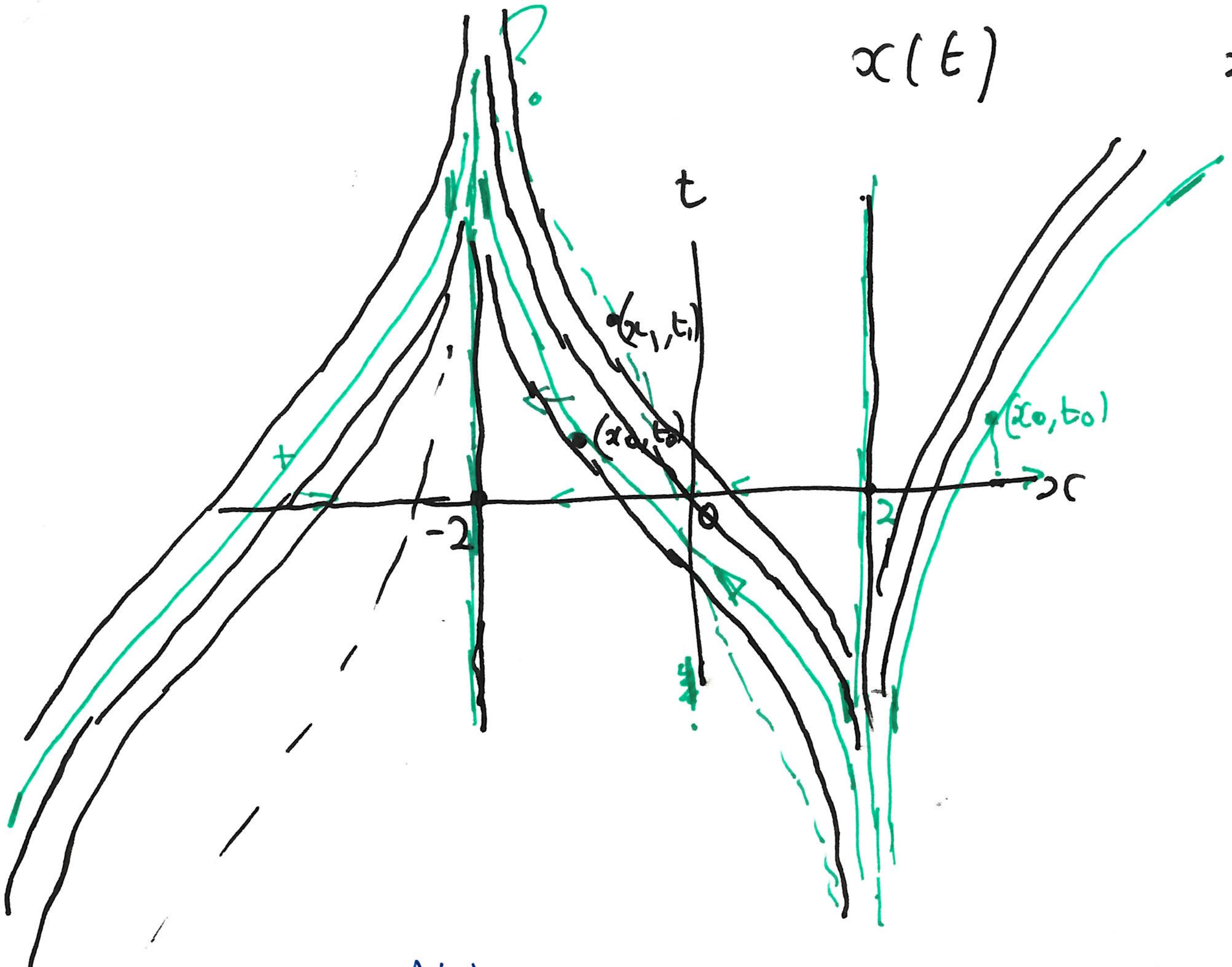
T.1.2

$$\frac{dx}{dt} = 4(x^2-4) \xrightarrow{\text{Calculus}} \int \frac{dx}{(x-2)(x+2)} = \int 4\,dt \quad \checkmark$$

$x(t)$ $\quad x = x_0, \; t = t_0.$ initial conds.

$x(t) \equiv 2$

try The calculus



Wolfram Alpha

streamplot (y, -x)

Example ~~2.1~~ 1.2.

$\dot{x} = y, \; \dot{y} = -x$

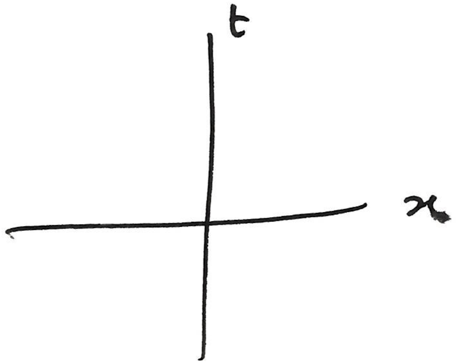
T1.3

Suppose $x(t)$ is a solution $\dot{x} = f(x)$

Then $x(t+c)$ is also a solution of $\dot{x} = f(x)$. } check that

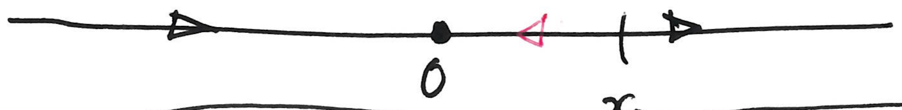
$x(t+c)$ is a translate of $x(t)$ in the t-dirn

Once you have sketched one curve between two fixed (two vertical lines) all the other solution curves are (t-)vertical translates



<u>Ex</u> $\dot{x} = x^2$

$$\left\{ \begin{array}{l} \dot{x} = x^4 \\ \quad = x^6 \\ \quad \vdots \end{array} \right.$$

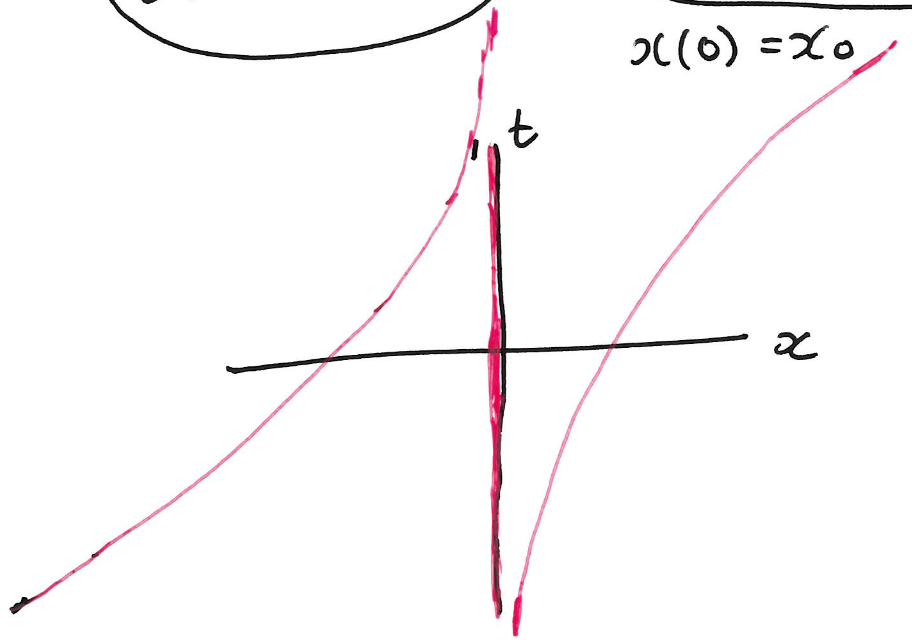


$t \to -\infty \quad x \to -\infty$
$t \to +\infty \quad x \to 0$

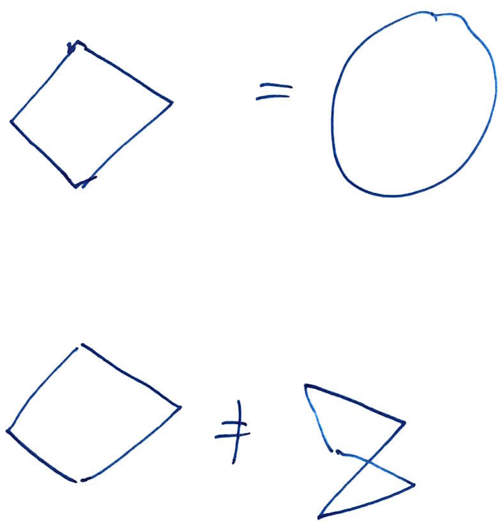
$t \to -\infty \quad x(t) \to 0.$
$t \to +\infty \quad x(t) \to \infty.$

$x(0) = x_0$

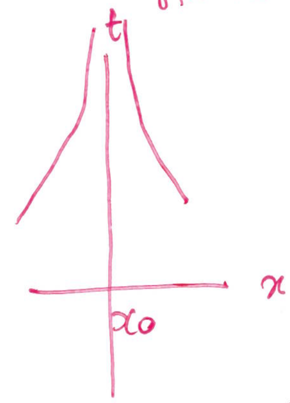
7.5
unstable fixed pnt



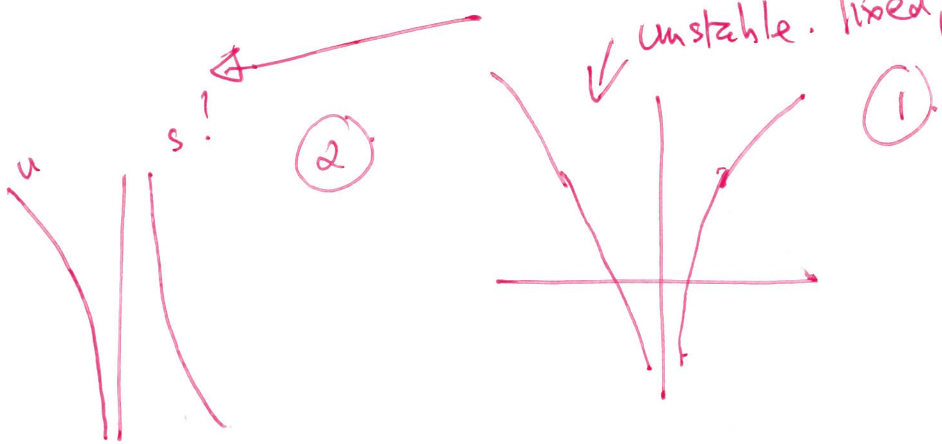
Top



stable fixed pt



unstable. fixed pt



$\dot{x} = -x$ stable

$\dot{x} = +x$ unstable.