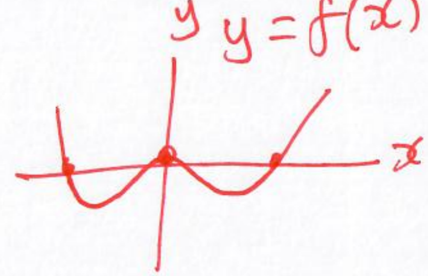
Lecture 22 - review of mock exam. $\boxed{22.1}$

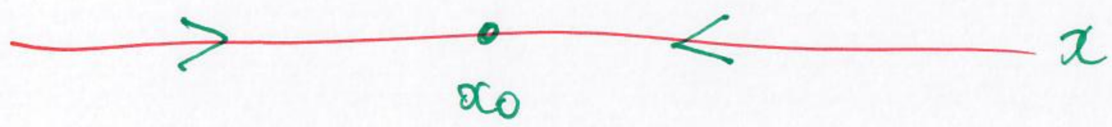
One dimensional systems

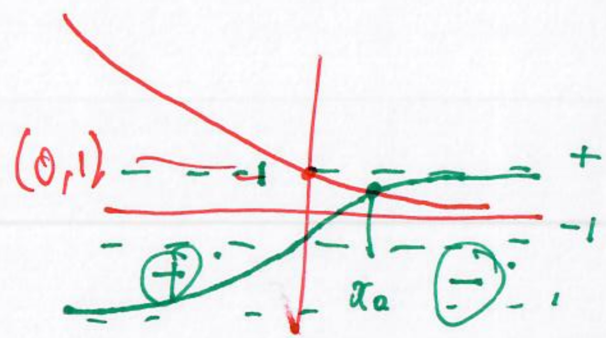
(a) $\dot{x} = f(x)$, $x \in \mathbb{R}$, examine the fn x + - & '0''s.

general discussion



(ii) $\dot{x} = \exp(-x) - \tanh(x)$





$\tanh(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$.

$\tanh(x)' = \operatorname{sech}^2(x) > 0$

$\boxed{22.2}$

(i) $\dot{x} = x^2 (x^6 + x^3 - 1)$ FP $x = 0$ (rep)

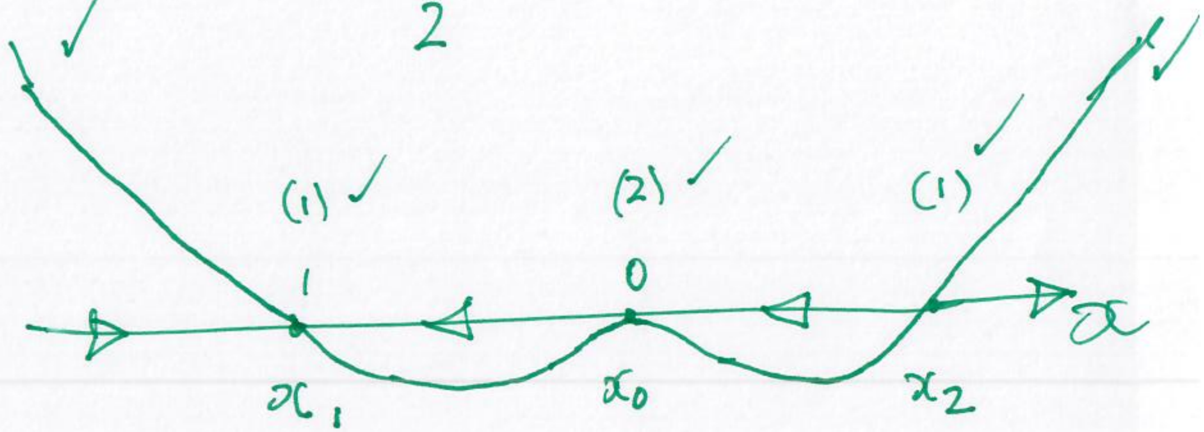
$\sim x^8$, large x

$$x^3 = \frac{-1 \pm \sqrt{1+4}}{2}$$

$$= \frac{-1 \pm \sqrt{5}}{2}$$

$$x_1^3 = \frac{-1 - \sqrt{5}}{2} < 0$$

$$x_2^3 = \frac{-1 + \sqrt{5}}{2} > 0$$

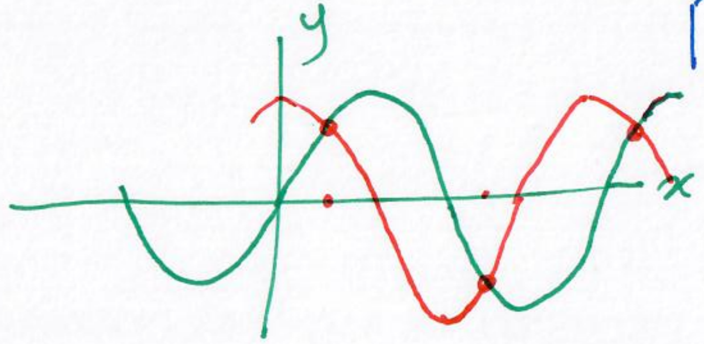


$x_1 < 0$, $x_2 > 0$

$x_0 = 0$ is zero!

(iii) $\dot{x} = \sin x - \cos x$

22.3



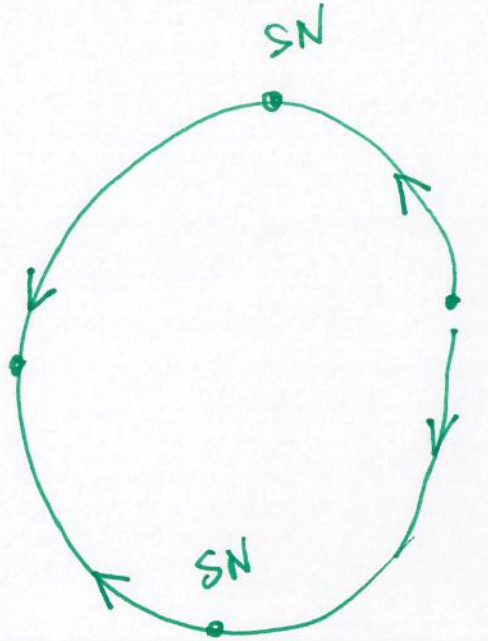
$\sin(A \mp B) = \sin A \cos B \mp \cos A \sin B$

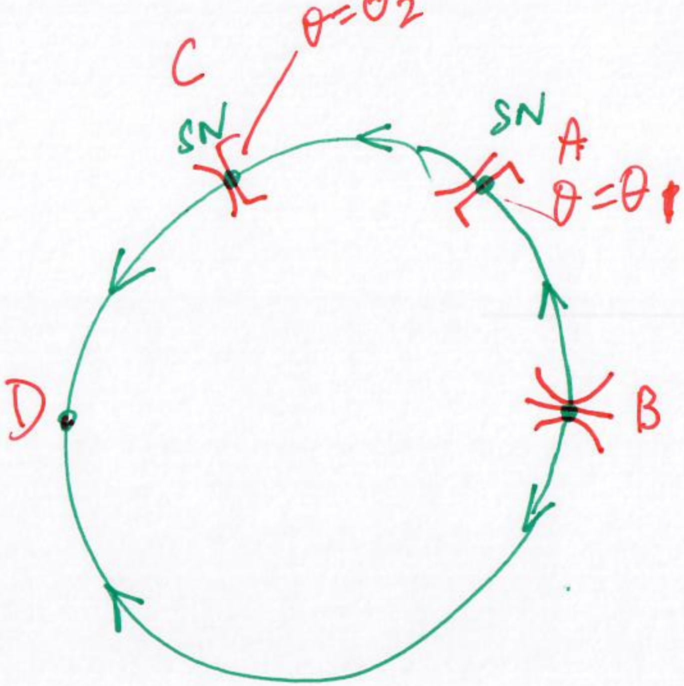
$$\dot{x} = \sin x - \cos x$$
$$= \sqrt{2}\left(\sin x \frac{1}{\sqrt{2}} - \cos x \frac{1}{\sqrt{2}}\right) = \sqrt{2}\left(\sin x \cos \pi/4 - \cos x \sin \pi/4\right) = \sqrt{2}\sin(x - \pi/4)$$

$\Rightarrow$ oscillators

$\boxed{22.4}$

(b)





$B(A) = [A, B)$.

$B(B) = \{B\}$.

$B(C) = [C, A)$

$B(D) = (B, C)$

(c)

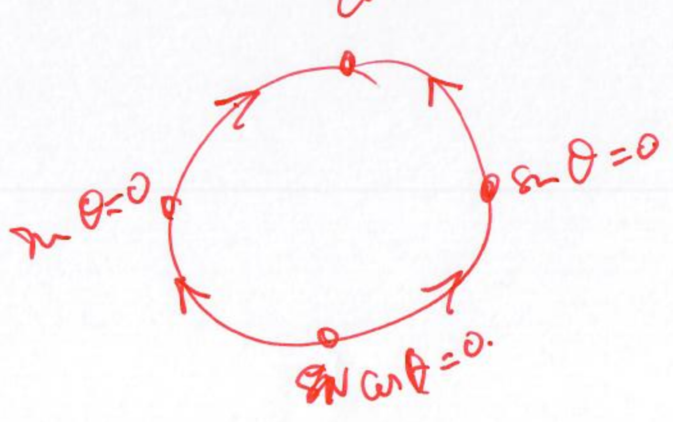
$\dot\theta > 0$ for $\theta \in (0, \pi) - \{A, C\}$

$\dot\theta < 0$ for $\theta \in (\pi, 2\pi)$

$$\overset{+ve}{}\quad\overset{+ve}{}$$

$\checkmark$ $\dot\theta = \sin\theta\,(1 - \cos\theta_1)(1 - \cos\theta_2)$

$0 < \theta_1, \theta_2 < \pi$



$\dot\theta = \cos\theta \sin^2\theta_1 \sin^2\theta_2$

$\sin^4\theta_1 \sin^4\theta_2$

22.5

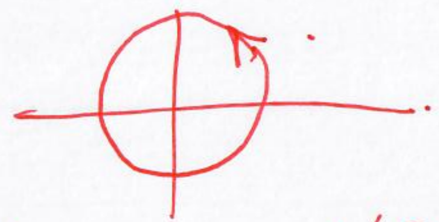
(d) $\dot{\theta} = f^4(\theta)$

$(f(\theta))^4$

$f : S^1 \to \mathbb{R}$ $\dot{\theta} \geq 0$

sequence of ~~coordi~~ ~~clockwise~~ S.N. with ~~clo~~ anti-clockwise flow between

$f(\theta) = c \neq 0$



anti-clockwise

$f(\theta) > 0 \quad \forall \theta$

$(f(\theta))^4 > 0 \quad \forall \theta$

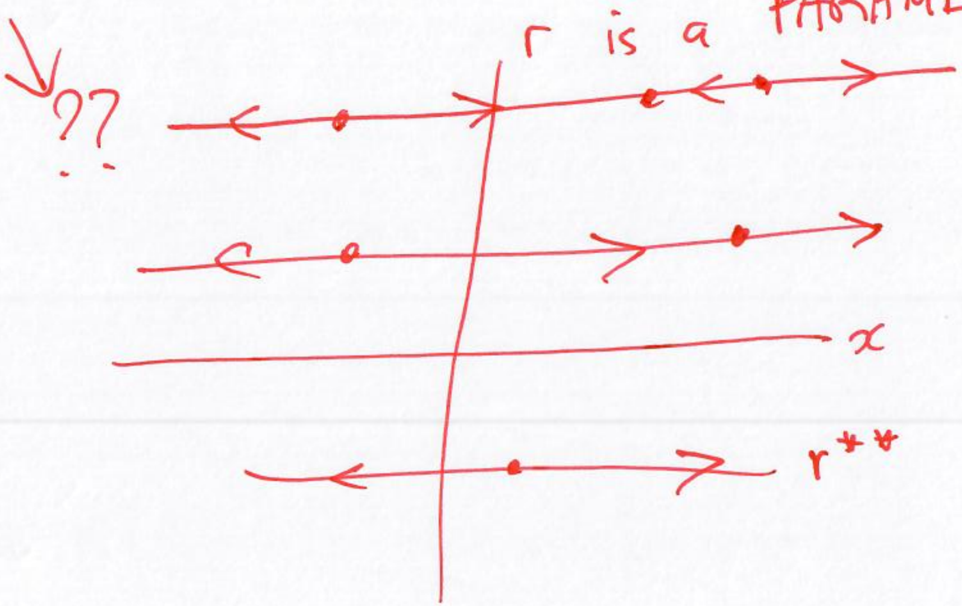
non-uniform circular flow.
anticlockwise

$f(\theta) = 0$ circle of fixed points

22.6

Q2 $\dot{x} = rx - x^2 + x^4$

Bifurcation diagram



$r = r^*$ $\dot{x} = r^* x - x^2 + x^4$

$r^* = 10$.

$\boxed{22.7}$

① Find the FPs in the (x, r) plane

$$\dot{x} = rx - x^2 + x^4$$

$$rx - x^2 + x^4 = 0$$

FPs of one dimensional (with a parameter!)

HORIZONTAL FLOW in charts ②?

$$\dot{r} \equiv 0$$

$$\rightarrow \quad rx = x^2 - x^4$$

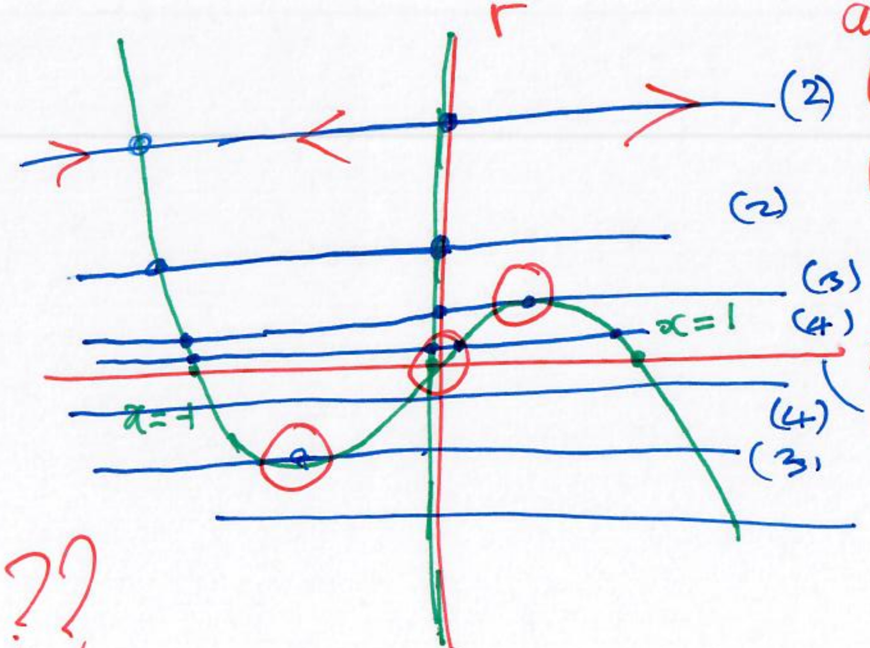
$$= x(r - x + x^3)$$

FPs $\quad x = 0, \ \forall r$

$$r = x - x^3$$

zero of r

$x = 0$, $x \neq \pm 1$



arrows to be added

bifurcations to be understood

3 bifurcation points??