x dependent variable
t independent variable. $\Big\}$ $x = x(t)$

$\frac{dx}{dt}$ ODE

$$\frac{dx}{dt} = x$$

$$\int \frac{dx}{x} = \int dt$$

$$\ln(x) = t + C$$

$$x = e^{t+C} = Ae^{t}$$

$A = e^{C}$.

$$\frac{dx}{dt} = x + \sin x$$

$$\int \frac{dx}{x + \sin x} = dt$$

$$\int \frac{1}{x + \sin x + e^{x}}\, dt. \rightarrow$$ Numerical ??

Starting cond

$x = x_0$, $t = t_0$ → potential "vaicly" in solutions.

$$\frac{d^2x}{dt^2} + x\frac{dx}{dt} + x^3 = 0$$

2nd order vs previous 1st order.

↓

1st order??

Def^n.

$$\left.\begin{aligned} \frac{dx}{dt} &= y \\ \frac{dy}{dt} &= \frac{d^2x}{dt^2} \end{aligned}\right\}$$

$$\frac{dy}{dt} + xy + x^3 = 0$$

$$\frac{dx}{dt} = y$$

$$\frac{dy}{dt} = -xy - x^3$$

1st order

2 dep variables

Ex $\frac{d^3x}{dt^3} + x\frac{d^2x}{dt^2} + \sin(x) = \frac{dx}{dt}$ (1)

$\frac{d^3x}{dt^3} = \cdots d^2$

$$\left.\begin{aligned} \frac{dx}{dt} &= y \\ \frac{dy}{dt} &= z \quad (2) \end{aligned}\right\} \text{dfn}$$

(3)

$$\frac{dz}{dt} = \frac{d^2y}{dt^2} = -xz - \sin x + y$$

$$\frac{dx}{dt} = y + x - \sin(z) = f_1(x,y,z)$$
$$\frac{dy}{dt} = y + \cos(z) + 0.x = f_2(x,y,z)$$
$$\frac{dz}{dt} = x + z = f_3(x,y,z)$$

NL

$$\dot{x} = y + x - z$$
$$\dot{y} = y + z$$
$$\dot{z} = x + z$$

Linear

$(x, y, z) \to (x_1, x_2, x_3)$

$$\frac{dx_1}{dt} = f_1(x_1, x_2, x_3)$$
$$\frac{dx_2}{dt} = f_2(x_1, x_2, x_3)$$
$$\frac{dx_3}{dt} = f_3(x_1, x_2, x_3)$$

$\underline{x} = (x_1, x_2, x_3)$

$\underline{f} : \mathbb{R}^3 \to \mathbb{R}^3$ (vector fn.)

$\underline{f}(\underline{x}) = (f_1(\underline{x}), f_2(\underline{x}), f_3(\underline{x}))$

$f_i : \mathbb{R}^3 \to \mathbb{R}$

$$\boxed{\dot{\underline{x}} = \underline{f}(\underline{x})}$$

n-dimensional case

$\underline{x} = (x_1, \dots, x_n)$, $\dot{\underline{x}} = \underline{f}(\underline{x})$, $\underline{x} \in \mathbb{R}^n$, $\underline{f} : \mathbb{R}^n \to \mathbb{R}^n$

First order DS in 2 variables

$$\dot{x} = y \quad , \quad \dot{y} = -x$$

$$\Rightarrow \quad \ddot{x} = \dot{y} = -x \qquad \text{SHM}$$

$$\frac{dx}{dt} = y \quad , \quad \frac{dy}{dt} = -x$$

$$\frac{dy}{dx} = \frac{-x}{y} \qquad y\,dy + x\,dx = 0 \qquad \Rightarrow \frac{x^2}{2} + \frac{y^2}{2} = C.$$

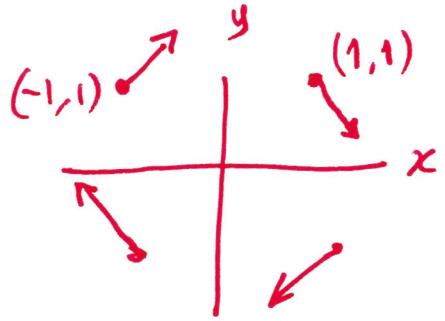
$$x^2 + y^2 = C \qquad x = x_0,\ y = y_0,\ t = 0$$

$x^2 + y^2 = x_0^2 + y_0^2$. (no time in this equation)

$x_0 = 0,\ y_0 = 0 \Rightarrow x(t) = y(t) = 0,\ \forall t.$

fixed point

Dalliance with 2 dims.



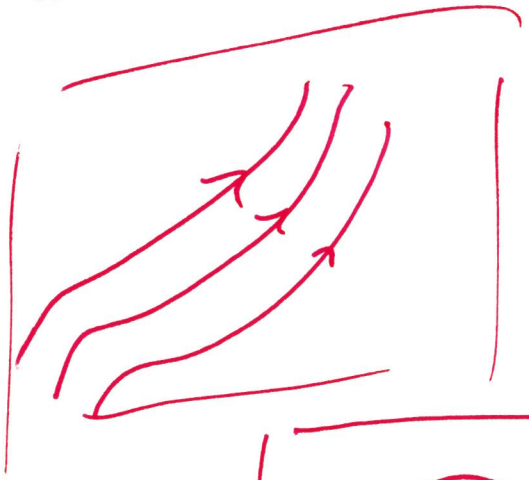
$x = 1,\ y = 1$

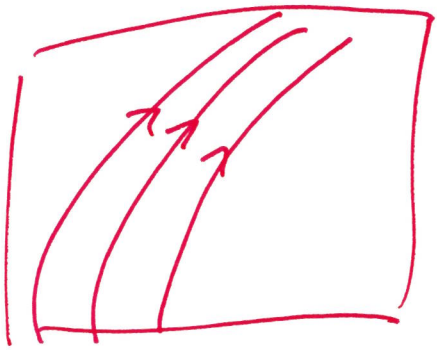
$\dot{x} = 1,\ \dot{y} = -1$

$(\dot{x}, \dot{y}) = (1, -1)$

1st order in 1 variable ODEs

$\mathbb{R}$, S ✓✓ L(2.1)





✓

×

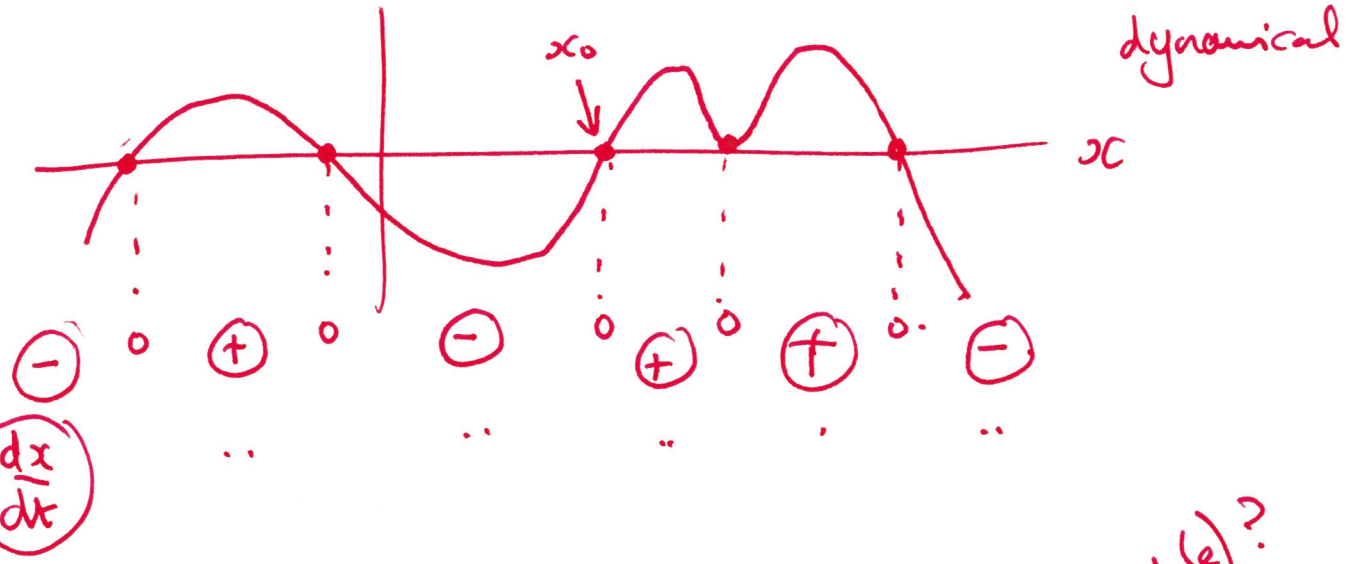
$\dot{x} = f(x)$, $f: \mathbb{R} \to \mathbb{R}$, $x = x(t)$, $t \in \mathbb{R}$

$f(x) = x$

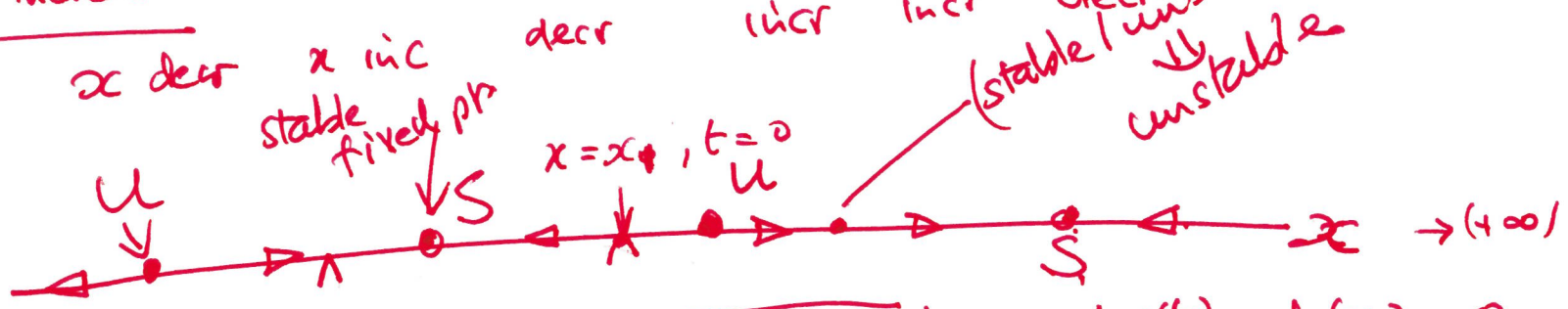
$f(x) = x + \sin x$?

$f(x) = x + \sin x + \cosh(x)$??

$\dot{x} = f(x)$



As t increases



$f(x_0) = 0$. Consider $x(t) = x_0$ $\Rightarrow$ $\frac{dx(t)}{dt} = \frac{d}{dt}(x_0) = 0$

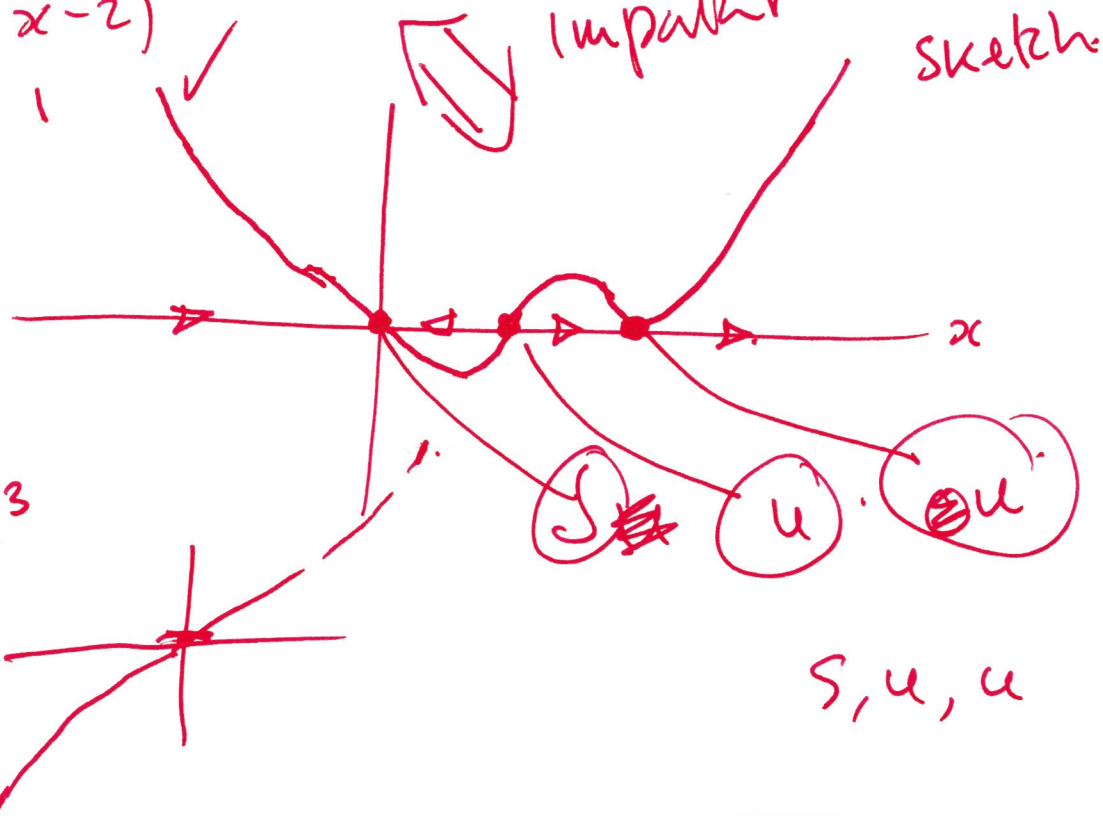
$f(x(t)) = f(x_0) = 0$

$\frac{d(x(t))}{dt} = f(x(t))$ ★

* zero of f $\Leftrightarrow$ fixed point of $\frac{dx}{dt} = f(x)$

Ex 1.2

$\dot{x} = x(x-1)(x-2)^2$



$x(x-2)$ x^3

S, U, U as x increases.

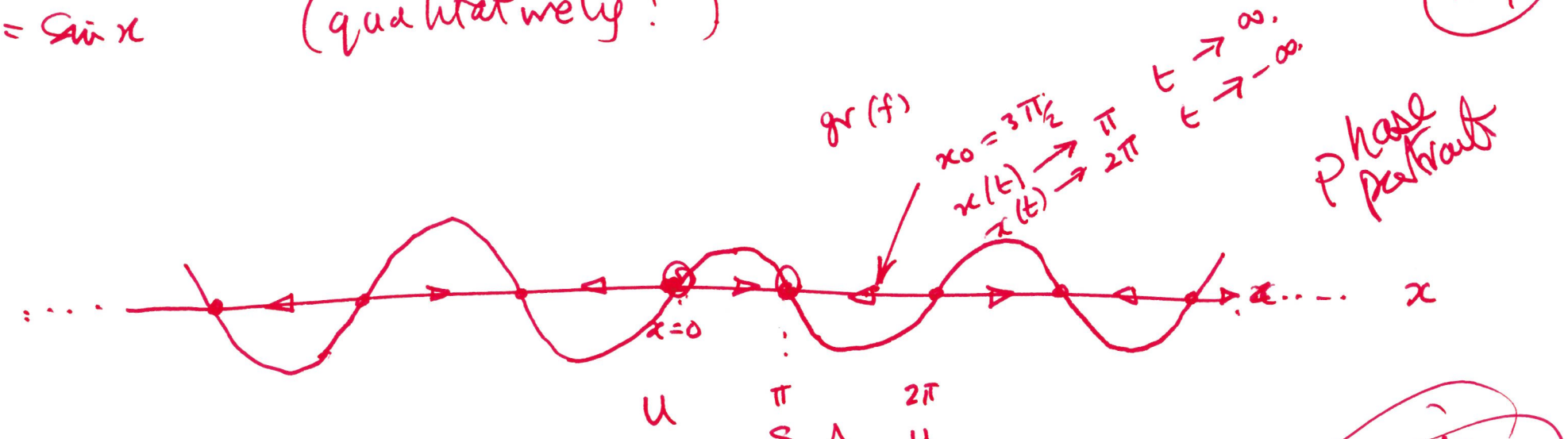
(2.3)

1.3 $\dot{x} = \sin x$

$$\int \frac{dx}{\sin x} = dt.$$

$$t = \ln\left(\frac{\csc(x_0) + \cot(x_0)}{\csc(x) + \cot(x)}\right)$$ Ex 1.10 p 5

$\dot{x} = \sin x$ (qualitatively!)



$x = n\pi$, fixed point

n = odd stable point

n = even unstable point.

$\dot{x} : t \; p(x)$