

SPA7010 Example Problems 2
(06 Feb 2020)

The Virial Theorem

Problem 2.1

State (without proof) the virial theorem for a self-gravitating stellar system, defining the terms used.

What are the conditions required for its application?

A galaxy cluster is observed to have a radius of 750 kpc and a velocity dispersion of 800 km s^{-1} . Use the virial theorem to estimate the total mass of the cluster, giving your answer in solar masses.

(You may assume the potential energy of a uniform sphere of mass M and radius R is $U = -3GM^2/(5R)$, where $G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$ is the gravitational constant, the mass of the Sun, $M_{\odot} = 2.0 \times 10^{30} \text{ kg}$, and 1 parsec (pc) = $3.09 \times 10^{16} \text{ m}$).

Problem 2.2

A star with mass m is executing a circular orbit with radius R and orbital velocity V around a stationary star with mass M .

Use the virial theorem to derive an expression for the period T of the orbital motion of mass m around M , showing that $T^2 \propto R^3$. Comment on your result.