Queen Mary
University of London

January Examination Period 2022-23

ECN322 Topics in Econometrics **Duration: 2 hours**

YOU ARE NOT PERMITTED TO READ THE CONTENTS OF THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR

Answer **three** questions. Questions 1 and 2 are **compulsory**, then **choose one** from the questions 3 and 4. If you answer more questions than specified, only the first answers (up to the specified number) will be marked. **JUSTIFY ALL YOUR ANSWERS.** Cross out any answer that you do not wish to be marked. There is a table of critical values from the standard normal distribution provided on page 7-8.

You are permitted to bring 20 x A4 pages of notes into your examination (i.e. 10 double sided pieces of paper). These can be typed or handwritten and can include graphs and images. They can include material from any source. Your notes must be stapled together and include your student ID number and the module code on the first page. You must submit your notes at the end of the examination with your answer booklet.

Calculators are permitted in this examination. Please state on your answer book the name and type of machine used. Complete all rough workings in the answer book and cross through any work that is not to be assessed.

Possession of unauthorised material at any time when under examination conditions is an assessment offence and can lead to expulsion from QMUL. Check now to ensure you do not have more than 20 pages of notes. You should also not have mobile phones, smart watches or unauthorised electronic devices on your person. If you do, raise your hand and give them to an invigilator immediately.

It is also an offence to have any writing of any kind on your person, including on your body. If you are found to have hidden unauthorised material elsewhere, including toilets and cloakrooms, it will be treated as being found in your possession. Unauthorised material found on your mobile phone or other electronic device will be considered an assessment offence. A mobile phone that causes a disruption in the exam is also an assessment offence.

EXAM PAPERS MUST NOT BE REMOVED FROM THE EXAM ROOM

Examiner: Yufei Li

Question 1 (30 marks)

Matrix A has the following spectral decomposition:

$$A = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} -2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & \frac{2}{3} \\ \frac{2}{3} & \frac{1}{3} & -\frac{2}{3} \\ \frac{2}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

Use his spectral decomposition to answer the following questions.

a) Write down the eigenvalues and corresponding unit eigenvectors of A.

[3 marks]

b) Find the trace and determinant of A.

[4 marks]

c) Find the rank of A.

[3 marks]

d) Is matrix A positive definite, positive semidefinite or neither?

[5 marks]

e) Is matrix A invertible? If yes, find the inverse matrix, otherwise provide your explanation.

[6 marks]

f) Could matrix A be a variance-covariance matrix of a 3×1 random vector?

[4 marks]

g) Find the expression of the spectral decomposition of A^3.

[5 marks]

Question 2 (30 marks)

a) Explain briefly the similarities and differences between the Monte Carlo simulation and bootstrap estimation methods.

[6 marks]

b) $X_1, X_2, \cdots, X_n$ is a sample of i.i.d. random variables; the sample mean is

$$\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

i) If X_i are normally distributed, what can you say about the distribution of the sample mean?

[3 marks]

ii) If X_i are not normally distributed, but the sample size is large, what can you say about the distribution of the sample mean?

[3 marks]

iii) If X_i are not normally distributed, for example $X_i \sim \chi^2(2)$, but the sample size is very small, how can you approximate the distribution of the sample mean? Specify your approach and briefly explain what you do at each step. (You **don't** need to write EViews code.)

[8 marks]

iv) Assume that Figure 1 is the histogram of the sample mean you obtained from iii) using EViews, and you also add the theoretical density line of normal distribution. Provide your comments on the distribution of the sample mean using Figure 1.

[3 marks]

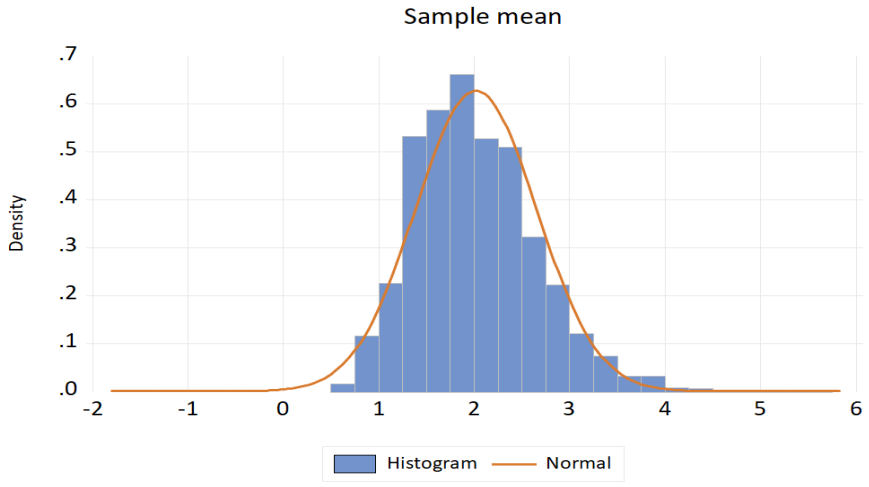


Figure 1: Histogram of sample mean

c) Assume that you do not know the distribution of X_i. For example, you are only given 10 observations,

$$3, 5, 1, -3, 8, 2, 4, -2, -1, 6.$$

i) You draw bootstrap samples to obtain information about the distribution of the sample mean. Choose **all** samples from the options below that are bootstrap samples.

[4 marks]

A. $1, 1, 3, 4, -1, 6, 6, 6, 3, 5$

B. $3, 5, -2, 1, 9, 2, 6, 8, 1, 2$

C. $-1, -2, 3, 2, -3, 6, 8, 5$

D. $2, 4, 1, 5, 6, 1, 1, 1, 2, 4$

ii) The bootstrap sample quantiles obtained using EViews are as follows:

1% quantile: -0.5	5% quantile: 0.45
10% quantile: 0.8	90% quantile: 3.7
95% quantile: 4.2	97.5% quantile: 4.4

Construct a 90% bootstrap confidence interval for the population mean.

[3 marks]

Question 3 (40 marks)

Linear mean regressions and linear quantile regressions of log wages on years of schooling and other factors such as race and potential labour market experience were run on a large number of data from three censuses from years 1980, 1990 and 2000. The coefficients on the years of schooling are reported in the table below. The numbers in the parentheses are the standard errors of the coefficients.

		Desc. Stats.		Quantile Regression Estimates					OLS Estimates	
Census	Obs.	Mean	SD	0.1	0.25	0.5	0.75	0.9	Coeff.	Root MSE
1980	65023	6.4	0.67	.074 (.002)	.074 (.001)	.068 (.001)	.070 (.001)	.079 (.001)	.072 (.001)	0.63
1990	86785	6.46	0.06	.112 (.003)	.110 (.001)	.106 (.001)	.111 (.001)	.137 (.003)	.114 (.001)	0.64
2000	97397	6.5	0.75	.092 (.002)	.105 (.001)	.111 (.001)	.120 (.001)	.157 (.004)	.114 (.001)	0.69

Table 1: Quantile regression coefficients for schooling

The dependent variable is *log wage*, explanatory variable is *years of schooling*, and other factors are *race, potential labour market experience*.

a) We include other factors in the regression to avoid what kind of bias?

[3 marks]

b) Which of the coefficients are significant at the 5% significance level? Justify your conclusion.

[8 marks]

c) Comment on the **within group wage inequality** in 1980, 1990 and 2000.

[6 marks]

d) Describe the change in **overall wage inequality** from 1980 to 1990.

[6 marks]

e) Calculate the percentage wage increase associated with an additional year of schooling for the lowest decile, median, and highest decile in the 2000 census, ? Also, make an interpretation of the OLS coefficient (0.114) in the same year.

[8 marks]

f) To what extend can the estimated mean effect reflect the effect on different quantiles in these regression?

[9 marks]

Question 4 (40 marks)

Consider the linear model

$$y_i = \beta_0 + \beta_1 x_{1i} + \cdots + \beta_k x_{ki} + \varepsilon_i,\ i = 1, \cdots, n$$

where $E(\varepsilon_i | x_{1i}, \cdots, x_{ki}) = 0$, and $\varepsilon_i \sim_{i.i.d.} N(0, \sigma^2)$.

a) Write this model in matrix form using $Y, \boldsymbol{\beta}, X, \boldsymbol{\varepsilon}$ and specify the dimension of each component.

[6 marks]

b) Derive the OLS estimator $\hat{\boldsymbol{\beta}}$ of $\boldsymbol{\beta}$.

[10 marks]

c) Denote $P_X = X(X'X)^{-1}X'$, $M_X = I_n - P_X$. Show that any $n \times 1$ vector z can be decomposed into two orthogonal parts by using P_X and M_X.

[8 marks]

In addition, if $E(\boldsymbol{\varepsilon}|X) = \mathbf{0}$, and $\boldsymbol{\varepsilon}|X \sim N(0, \Omega)$, answer the following questions.

d) Show that $\hat{\boldsymbol{\beta}}$ is an unbiased estimator of $\boldsymbol{\beta}$.

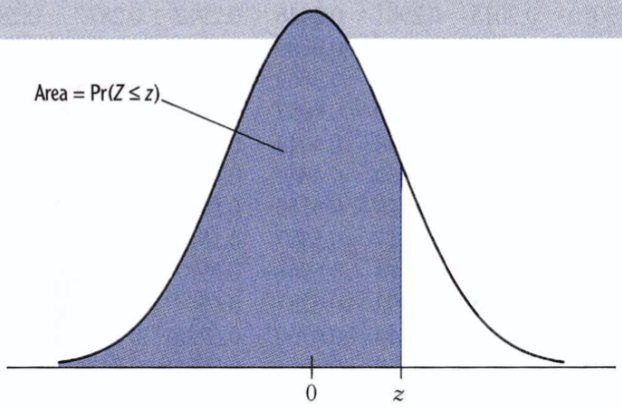
[6 marks]

e) Compute the conditional distribution of $\hat{\boldsymbol{\beta}}$.

[10 marks]

End of Paper - An appendix of 2 pages follows

TABLE 1 The Cumulative Standard Normal Distribution Function, $\Phi(z) = \Pr(Z \leq z)$



	Second Decimal Value of z									
z	0	1	2	3	4	5	6	7	8	9
–2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
–2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
–2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
–2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
–2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
–2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
–2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
–2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
–2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
–2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
–1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
–1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
–1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
–1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
–1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
–1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
–1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
–1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
–1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
–1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
–0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611

continued on next page

TABLE 1 *(continued)*

	Second Decimal Value of z									
z	0	1	2	3	4	5	6	7	8	9
−0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
−0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
−0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
−0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
−0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
−0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
−0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
−0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
−0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986

This table can be used to calculate $\Pr(Z \leq z)$ where Z is a standard normal variable. For example, when $z = 1.17$, this probability is 0.8790, which is the table entry for the row labeled 1.1 and the column labeled 7.

End of Examination/ Yufei Li