

Main Examination Period 2023

ECN211 Microeconomics II Duration: 2 hours

# YOU ARE NOT PERMITTED TO READ THE CONTENTS OF THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR

# Answer each of the FOUR questions

Calculators are permitted in this examination. Please state on your answer book the name and type of machine used. Complete all rough workings in the answer book and cross through any work that is not to be assessed.

Possession of unauthorised material at any time when under examination conditions is an assessment offence and can lead to expulsion from QMUL. Check now to ensure you do not have any notes, mobile phones, smartwatches or unauthorised electronic devices on your person. If you do, raise your hand and give them to an invigilator immediately.

It is also an offence to have any writing of any kind on your person, including on your body. If you are found to have hidden unauthorised material elsewhere, including toilets and cloakrooms it will be treated as being found in your possession. Unauthorised material found on your mobile phone or other electronic device will be considered the same as being in possession of paper notes. A mobile phone that causes a disruption in the exam is also an assessment offence.

## EXAM PAPERS MUST NOT BE REMOVED FROM THE EXAM ROOM

**Examiners:** Paweł Dziewulski

Page 2 ECN211 (2023)

### Question 1

Consider an economy with two goods s=1,2 and two consumers i=A,B with identical preferences. Whenever an individual consumes  $x_1$  units of good 1 and  $x_2$  units of good 2, their utility is given by

$$u(x_1, x_2) = \frac{1}{2} \ln x_1 + \frac{1}{2} \ln x_2.$$

(i) Calculate the marginal rate of substitution for those preferences at any consumption profile  $(x_1, x_2)$ . What is the marginal rate of substitution when  $x_1 = x_2$ ?

[4 marks]

Suppose that consumer A is endowed with 2 units of good 1, but has no endowment of good 2. Consumer B has no endowment of good 1, but owns 4 units of good 2. In the remainder of this question, let p denote the price of good 1 and let the price of good 2 be normalised to 1.

(ii) Show that, given price p, the demand of A for good 1 and 2 are given by  $x_1^A=1$ ,  $x_2^A=p$ , respectively. Similarly, show that the demand of B is given by  $x_1^B=2/p$  and  $x_2^B=2$ .

[6 marks]

(iii) Show that p=2 is the competitive equilibrium price.

[4 marks]

(iv) Plot the above economy and equilibrium in an Edgeworth box. Make sure to include the endowments, equilibrium prices, allocations, and sketch some indifference curves for each agent. In addition, include the  $45^\circ$  line for each consumer.

[5 marks]

Suppose we interpret the economy above as follows. There are two states of the world s=1,2, each equally likely. Let goods 1 and 2 correspond to the consumption of each agent in the respective state.

(v) The government finds it imperative that (in this economy) each consumer should be fully insured in equilibrium. That is, agent A should have the same level of consumption in state 1 and 2, and the same should apply to consumer B. Is it possible to reach this objective by making appropriate transfers of the initial endowment between the two consumers? Precisely motivate your answer.

[6 marks]

**ECN211 (2023)** Page 3

#### **Question 2**

A profit-maximising firm has to decide how much pollution to emit. Its profit depends on the level of emission h and the amount of money  $w_f$ , and is given by

$$\pi(h) = w_f + \theta h - \frac{1}{2}h^2,$$

where  $\theta$  is a positive parameter. The pollution affects a consumer living nearby. The consumer's payoff depends on the level of emissions h and the amount of money  $w_c$  she has. Her preferences are

$$\phi(h) = w_c - \frac{1}{2}h^2.$$

(i) What is the profit-maximising level of pollution for the firm (as a function of  $\theta$ )?

[6 marks]

(ii) What is the Pareto optimal level of pollution? How does it depend on the parameter  $\theta$ ? How does the optimal level of pollution relate to the one selected by the firm?

[7 marks]

(iii) Obtain an expression for the optimal Pigouvian tax as a function of  $\theta$ . From the welfare perspective, would the economy be better off if the government introduced the optimal level of quota on pollution? Precisely motivate your answer.

[6 marks]

Suppose the government does not know the actual value of  $\theta$ , but they know that it may obtain the value  $\theta_L = 2$  or  $\theta_H = 4$  with equal probability.

(iv) The government needs to decide whether to implement Pigouvian taxation or quota before the uncertainty regarding  $\theta$  is resolved. Assuming that the government is interested in minimising the expected dead weight loss, which policy would be better? [Hint: By symmetry, it suffices to compare the dead weight loss induced by each policy, conditional on  $\theta = \theta_L = 2$ .]

[6 marks]

Page 4 ECN211 (2023)

### **Question 3**

(i) What is the *certainty equivalent* of a lottery? How can it be used to determine whether or not someone should participate in one?

[4 marks]

Agents A and B are risk averse expected utility maximisers, each deriving utility  $u(w) = \ln w$  from their final wealth w. There is a lottery that would leave A with final wealth of either b or b0, each equally likely; another lottery that would leave b1 with final wealth of either b3 or b4, again, each equally likely.

(ii) Assuming that both agents have the same level of initial wealth, explain why they would either both accept or decline their separate gambles.

[6 marks]

In the remainder of the question, assume that each agent has the initial wealth equal to 12.

(iii) If the two lotteries are independent, would you advise them to each participate in their lottery and share the costs and proceeds equally? Precisely motivate your answer.

[5 marks]

(iv) Suppose that the two lotteries are perfectly negatively correlated. That is, one lottery yields high prize if, and only if, the other one yields the low one. Would you advise them to each participate in their lottery (again, sharing the costs and proceeds equally)? Does your answer change if the utility of the agents was not given by  $u(w) = \ln w$ ?

[5 marks]

(v) Finally, suppose that the two lotteries are independent with probability p, and perfectly negatively correlated with the remaining probability. For what value of p would you advise them to each participate in their lottery (again, sharing the costs and proceeds equally)?

[5 marks]

**ECN211 (2023)** Page 5

#### **Question 4**

A risk-neutral principal (P) must delegate one task to a risk-neutral agent (A). The agent can exert two levels of effort: e=0 and e=1. There are two possible outcomes (revenue levels) for the principal:  $\pi=0$  and  $\pi=8$ . The principal can offer a contract  $(w_L,w_H)$ , where  $w_l$  denotes a monetary transfer from the principal to the agent if the principal's outcome is equal to 0, and  $w_H$  is a monetary transfer from the principal to the agent whenever  $\pi=8$ . The principal maximises the expected profit equal to the difference between the expected revenue  $\mathbb{E}\pi$  and the expected wage bill  $\mathbb{E}w$ .

The agent's utility is given by  $\mathbb{E}w-\psi(e)$ , where  $\psi(e)$  is the disutility from effort e. Assume throughout that  $\psi(0)=0$  and  $\psi(1)=3$ . Whenever the agent exerts effort e=0, the probability of high revenue  $\pi=8$  is equal to 1/3. If e=1, the probability of the same event is 2/3. If the agent does not accept the contract, they receive the utility of 0 from their outside option.

(i) First, suppose that the principal can perfectly observe the effort level of the agent. What would be the optimal contract if they wanted to induce no effort, i.e., e=0? What would be the optimal contract is they wanted to induce high effort, i.e., e=1?

[7 marks]

In the remainder of the question assume that the principal is unable to observe the effort of the agent.

- (ii) What would be the optimal contract if the principal wanted to induce no effort, i.e., e=0? [5 marks]
- (iii) What would be the optimal contract if the principal wanted to induce high effort, i.e., e=1? (You may assume that negative transfers are possible.)

[7 marks]

(iv) From the point of view of the principal, what is the optimal level of effort when actions of the agent are not observable? Precisely motivate your answer.

[6 marks]