January Examination Period 2022-23

ECN-115—Mathematical Methods in Economics and Finance ***Duration: 2 hours***

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Answer ALL questions

Calculators are not permitted in this examination. Complete all rough workings in the answer book and cross through any work that is not to be assessed.

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Examiner: Evgenii Safonov

Question 1

Describe the Principle of Mathematical Induction.

[10 marks]

Question 2

Calculate the following limit:

$$\lim_{n\to\infty}\left(\frac{(2n+1)(3n+2)}{n^2+(n-1)(n+4)}\right)$$

[5 marks]

Question 3

Calculate the derivative of the function $f:(0,\infty)\to\mathbb{R}$ at point x_o.

a) When the function f is given by

$$f(x) \ = \ \frac{x^2+1}{x}$$

at point $x_o = 2$.

[5 marks]

b) When the function f is given by

$$f(x) \ = \ x^{ln(x)}$$

at point $x_o = e$, where $e = 2.71828...$ is the Euler's number (the base of the natural logarithm).

[7 marks]

Question 4

a) State the Fundamental Theorem of Calculus (You may use your own way to describe the theorem, but try to be as formal as you can).

[9 marks]

b) Calculate the following integral:

$$\int_0^4 \frac{1}{1+3\sqrt{x}}\,dx$$

[15 marks]

Question 5

Consider a two-variable function $f:(0,1)\times(0,1)\to\mathbb{R}$ (thus, the variables x, y can take values such that $0<x<1$ and $0<y<1$) given by

$$f(x,y) \ = \ \ln\left(x^2\cdot(1-y)\right)$$

Calculate the partial derivatives $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at an arbitrary point of the domain of the function f. Simplify your answers when possible.

[8 marks]

Question 6

Consider a function $f : \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} e \cdot x - e^x & \text{if} \quad x \leq a \\ \dfrac{x^2}{2} + \dfrac{e-4}{2} - \sqrt{e} & \text{if} \quad x > a \end{cases} \tag{1}$$

where $e = 2.71828...$ is the Euler's number (the base of the natural logarithm), and the parameter a is such that $a \in [0, 2]$.

a) Are the following propositions true? Provide (short, to save your time) arguments why or why not.

  i) For any value of the parameter $a \in [0, 2]$, the function f is continuous (that is, continuous at all points $x \in \mathbb{R}$).

[4 marks]

  ii) The function f is differentiable at all points x such that $x \in (-\infty, a) \cup (a, \infty)$.

[4 marks]

Suppose now that we want to maximise the function f given by eq. (1) over the set $X = [0, 2]$.

b) State the Weierstrass Theorem that is related to the analysis of the problem (a version that has been studied in the lectures; but if you formulate a more general version, it is also fine).

[7 marks]

c) Can we use the Weierstrass Theorem to argue that $\max\limits_{x \in [0,2]} f(x)$ exists? Why or why not?

[3 marks]

d) Find $\max\limits_{x \in [0,2]} f(x)$ (if it exists). Your answer should, in general, depend on the parameter $a \in [0, 2]$. Hint: you may analyse two auxiliary maximisation problems: maximising the function f over the set $[0, a]$, and maximising f over $(a, 2]$. It might be useful to note that $e \cdot \frac{1}{2} - e^{\frac{1}{2}} = \frac{2^2}{2} + \frac{e-4}{2} - \sqrt{e}$. Note also that $\sqrt{e} = 1.6487...$, and $e/2 = 1.3591...$

[20 marks]

e) Is there a value of the parameter $a \in [0, 2]$ such that there are two maximisers of the function f over the set $X = [0, 2]$ (that is, there are two points at which function f attains its maximum over the set X)? If yes, what is this value?

[3 marks]

End of Paper

End of Examination/ Evgenii Safonov