Queen Mary
University of London

MSc Examination by Course Unit

Tuesday 21st May 2013 18:15 - 21:15

ASTM002 The Galaxy Duration: 3 hours

YOU ARE NOT PERMITTED TO READ THE CONTENTS OF THIS QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY AN INVIGILATOR.

Instructions:

Answer ALL questions from Section A. Answer TWO questions from Section B. Section A carries 50 marks, each question in section B carries 25 marks.

If you answer more than two questions from Section B, only the best two will count, except for the award of a bare pass.

Only non-programmable calculators are permitted in this examination. Please state on your answer book the name and type of machine used.

Complete all rough workings in the answer book and cross through any work that is not to be assessed.

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Examiners:
Dr. W. Sutherland, Prof. J. Emerson.

Useful Information

In this paper, π and e represent the standard mathematical constants.
G is the gravitational constant, with $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}.
c is the velocity of light, with $c = 3.00 \times 10^8$ m s^{-1}.
1 parsec (pc) $= 3.09 \times 10^{16}$ m.
1 astronomical unit (AU) $= 1.50 \times 10^{11}$ m.
The mass of the Sun is $M_\odot = 2.0 \times 10^{30}$ kg.
The distance of the Sun from the Galactic Centre is $R_0 = 8.0$ kpc.
Poisson's equation states that $\nabla^2\Phi = 4\pi G\rho$ at any point in a gravitational field, where Φ is the gravitational potential, G is the constant of gravitation, and ρ is the mass density at that point.

The Laplacian of a scalar function Φ in a spherical coordinate system (r, θ, ϕ) is

$$\nabla^2\Phi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\,\frac{\partial\Phi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\,\frac{\partial\Phi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\Phi}{\partial\phi^2} \ .$$

The Laplacian of a scalar function Φ in a cylindrical coordinate system (R, ϕ, z) is

$$\nabla^2\Phi = \frac{1}{R}\frac{\partial}{\partial R}\left(R\,\frac{\partial\Phi}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2\Phi}{\partial\phi^2} + \frac{\partial^2\Phi}{\partial z^2} \ .$$

The Jeans equations in a steady-state (i.e. time-independent), spherically-symmetric galaxy give the following result

$$\frac{\mathrm{d}}{\mathrm{d}r}\Big(n\,\langle v_r^2\rangle\Big) + \frac{n}{r}\left[\,2\langle v_r^2\rangle - \langle v_\theta^2\rangle - \langle v_\phi^2\rangle\right] = -\,n\,\frac{\mathrm{d}\Phi}{\mathrm{d}r} \ ,$$

in spherical coordinates, where n is the number density of stars at a distance r from the centre, v_r, v_θ and v_ϕ are the components of the velocity in the r, θ and ϕ directions, and $\Phi(r)$ is the gravitational potential.

In the absence of cosmological effects, the apparent magnitude m of an astronomical object in a photometric band is related to its absolute magnitude M in that band and its distance D from the observer by

$$m - M = 5\log_{10}(D/\mathrm{pc}) - 5 + A \ ,$$

where A is the extinction in the same band expressed in magnitudes.

Oort's constants within the Galaxy are defined as

$$A \equiv \frac{1}{2}\left(\frac{\langle v_\phi\rangle}{R} - \frac{\partial\langle v_\phi\rangle}{\partial R}\right) \qquad \text{and} \qquad B \equiv -\frac{1}{2}\left(\frac{\langle v_\phi\rangle}{R} + \frac{\partial\langle v_\phi\rangle}{\partial R}\right) \ ,$$

where $\langle v_\phi\rangle$ is the mean tangential velocity in the Galactic disc, and R is the distance from the Galactic Centre, and the above are evaluated at $R = R_0$.

SECTION A **Answer ALL questions in Section A**

Question A1

Discuss and contrast the observed properties of spiral and elliptical galaxies: refer to their morphologies, colours, spectra, gas and dust content, and stellar populations.

[6 marks]

Question A2

Explain the difference between dissipational and dissipationless collapse in galaxy formation. Explain why gas is likely to settle to a rotating disc during galaxy formation.

[4 marks]

Question A3

Define the terms “strong encounter” and “weak encounter” in relation to an encounter between two stars in a large stellar system.
Show that in a weak encounter between two stars of mass m, the change δv in velocity v of one star in the reference frame of the other is given by

$$\delta v = \frac{2Gm}{bv}$$

where G is the gravitational constant and b is the impact parameter. (You may assume that the deflection angle is small, and you may quote the result $\int_{-\infty}^{\infty} (k_1 + k_2 s^2)^{-3/2}\, ds = 2/(k_1\sqrt{k_2})$) .

[6 marks]

Question A4

State (without proof) the virial theorem for a self-gravitating stellar system. What are the conditions required for its application ?
Using the virial theorem, show that for a stellar system of radius R containing N stars each of mass m, the typical velocity is given by $v \approx \sqrt{GNm/R}$. (You may assume that the potential energy for a uniform sphere of mass M and radius R is approximately $-GM^2/R$).

[5 marks]

Question A5

An “integral of motion” is a function of position and velocity which is conserved along a star’s orbit.

a) Prove that specific energy $E_m = \frac{1}{2}v^2 + \Phi(\mathbf{x})$ is an integral of motion for any time-independent gravitational potential.

[2 marks]

b) Prove that the component of angular momentum around the $z-$axis, L_z, is an integral of motion for a time-independent potential which is axisymmetric around this axis.

[3 marks]

Question A6

Explain the meaning of the terms HI, HII and H_2 referring to hydrogen in the Galaxy, and describe the main observational methods used to probe the distribution of each species, and the typical environments where they are found.

[4 marks]

Question A7

In Galactic chemical evolution, the symbols X, Y, Z denote the mass fractions of hydrogen, helium and heavy elements respectively. Give typical values for these in the ISM. What processes produce most of the helium, and most of the heavy elements ?

[4 marks]

Question A8

A star near the Galactic plane is observed to have a Sun-like spectrum, and apparent magnitudes in the blue and visual bands of $B = 17.47$, $V = 16.42$. Given that the Sun has absolute magnitudes in these bands of $M_B = 5.48$, $M_V = 4.83$, and the reddening ratio for interstellar dust is $A_V/E(B-V) = 3.0$, estimate (i) the reddening of the star, (ii) the V-band extinction in magnitudes, and (iii) the distance to the star.

[4 marks]

Question A9

General Relativity predicts that the bending angle α (assumed small) for a light ray passing a distance b from a compact object of mass M is given by

$$\alpha = \frac{4GM}{c^2 b} \ .$$

Given this, considering a gravitational lens system where the Earth, lens and source are exactly collinear, show that the physical Einstein ring radius is

$$r_E = \sqrt{\frac{4GM}{c^2}\frac{D_{LS}D_L}{D_S}} \quad ,$$

where M is the lens mass, and D_L is the distance from Earth to the lens, D_S is the distance from Earth to the source, and D_{LS} is the distance between lens and source.

[6 marks]

Question A10

Describe the general features of the components of the Milky Way Galaxy, including the disc, bulge/bar, stellar halo and dark matter halo.

[6 marks]

SECTION B **Answer TWO questions from Section B**

Question B1

a) The fundamental plane for elliptical galaxies has the form

$$R_S I_0^{0.8} \sigma_0^{-1.3} = \text{constant} ,$$

where R_S is a characteristic radius, I_0 is central surface brightness and σ_0 is line-of-sight velocity dispersion. Show that this implies luminosity $L \propto \sigma^{2.6}/I_0^{0.6}$.

[3 marks]

b) Explain how the above quantities are determined observationally, and used to estimate relative distances to galaxy clusters.

[4 marks]

c) Discuss the main properties and significance of dust in the interstellar medium of the Galaxy.

[4 marks]

d) Explain the term *asymmetric drift* for stars in the Solar neighbourhood, and discuss how it varies with stellar age.

[4 marks]

e) The disk of the Milky Way may be modelled as a double-exponential disk with stellar density given by

$$\rho(R, z) = \rho_0 \, e^{-|z|/h} \, e^{-(R-R_0)/L} ,$$

where R, z are Galactocentric cylindrical coordinates, ρ_0 is the local stellar density, h is the scale height and L is the scale length. Observations imply $\rho_0 \simeq 0.1 M_\odot \, pc^{-3}$, $h \simeq 250 \, pc$, $L \simeq 3 \, kpc$ and $R_0 \simeq 8 \, kpc$. Calculate the total stellar mass of the disk in this model.

[6 marks]

f) Discuss the main observational methods, and uncertainties, when estimating the disk stellar mass from the model above.

[4 marks]

Question B2

a) The continuity equation for the distribution function f of stars in the six-parameter phase space $(x_1, x_2, x_3, v_1, v_2, v_3)$ of position $\mathbf{x}$ and velocity $\mathbf{v}$ states that

$$\frac{\partial f}{\partial t} + \sum_{j=1}^{3} \left(\frac{\partial}{\partial x_j} \left(f \frac{dx_j}{dt} \right) + \frac{\partial}{\partial v_j} \left(f \frac{dv_j}{dt} \right) \right) = 0 \ ,$$

where t is time.

Derive the collisionless Boltzmann equation

$$\frac{\partial f}{\partial t} + \sum_{j=1}^{3} \left(\frac{dx_j}{dt} \frac{\partial f}{\partial x_j} + \frac{dv_j}{dt} \frac{\partial f}{\partial v_j} \right) = 0$$

from the continuity equation, showing your working.

[6 marks]

b) Describe the advantages of the Jeans equations relative to the collisionless Boltzmann equation for describing the distribution of stars in observed galaxies.

[2 marks]

c) Derive the second of the Jeans equations,

$$\frac{\partial (n \langle v_i \rangle)}{\partial t} + \sum_{j=1}^{3} \frac{\partial}{\partial x_j} \Big(n \langle v_i v_j \rangle \Big) = - \frac{\partial \Phi}{\partial x_i} n \ ,$$

from the collisionless Boltzmann equation above, where n is the number density of stars and $\langle v_i \rangle$ is the mean value of the v_i velocity component at a point. (Explain your definitions, working and assumptions).

[10 marks]

d) Assume that the Galactic halo is spherical, that it has no net rotation, that its velocity dispersion is isotropic and constant, that it has a potential $\Phi(r) = v_0^2 \ln(r/a)$ where v_0, a are constants, and that it has a stellar density profile of the form $n(r) \propto r^{-l}$ where l is a constant. Using the Jeans equation from the Useful Information above, derive an expression for the velocity dispersion σ of halo stars in the solar neighbourhood in terms of v_0 and l.
If $v_0 = 220 \ \mathrm{km\,s^{-1}}$, calculate σ for a realistic value of l.

[7 marks]

Question B3

a) A galaxy is modelled using a spherically-symmetric gravitational potential of the form

$$\Phi(r) = -\frac{4\pi Gk}{r}\ln\left(\frac{r+a}{a}\right) ,$$

where r is the radial distance from the centre of the galaxy, a and k are constants and G is the constant of gravitation. Using Poisson's equation $\nabla^2\Phi = 4\pi G\rho$, show that the mass density ρ as a function of distance r implied by this potential is

$$\rho(r) = \frac{k}{r(r+a)^2} .$$

(You may quote a suitable equation from the Useful Information above).

[8 marks]

b) Show that in the above model, the mass $M(r)$ interior to a radius r is

$$M(r) = 4\pi k\left[\ln\left(\frac{r}{a}+1\right) - \frac{r}{r+a}\right]$$

Derive the asymptotic behaviour of the rotation curve $v_{circ}(r)$ in the limits $r \ll a$ and $r \gg a$.

[5 marks]

c) The distance l from the Milky Way to M31 (the Andromeda galaxy) is assumed to satisfy the differential equation

$$\frac{d^2l}{dt^2} = -\frac{GM}{l^2}$$

with the constant M being the total mass of the Local Group. Verify that

$$t = \tau_0(\eta - \sin\eta) ,$$
$$l = \left(GM\tau_0^2\right)^{\frac{1}{3}}(1-\cos\eta)$$

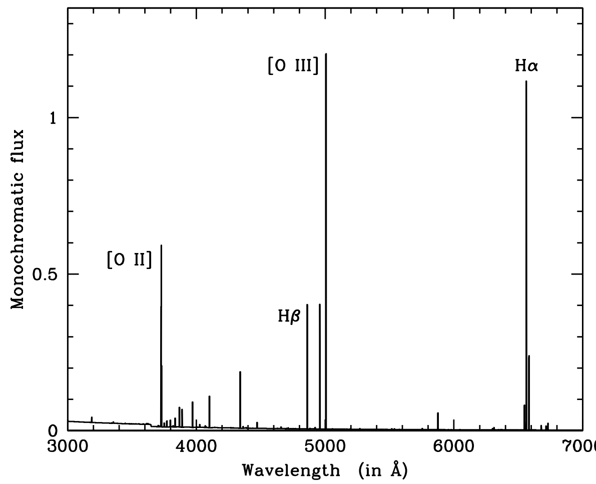
with τ_0 a constant, and η a parameter, is a solution of the above differential equation.

[6 marks]

d) Describe the observational evidence for dark matter in the Galaxy, and the difficulties in measuring the total amount; also, discuss the observational evidence that the dark matter is primarily non-baryonic.

[6 marks]

Question B4



a) The figure (above) shows the visible-wavelength spectrum of the Orion Nebula. Briefly explain the different emission lines labelled, and the physical mechanisms responsible for producing them.

[4 marks]

b) Explain, based on the energy levels of the hydrogen atom, why an HII region emits approximately one Balmer photon for each Lyman-continuum photon ($\lambda < 912\mathring{A}$) emitted by hot stars inside it.

[4 marks]

c) Explain briefly the different properties of Type Ia and Type II supernovae with respect to galactic chemical evolution. Which chemical elements are mostly synthesised by each type ?

[2 marks]

d) List any four assumptions behind the Simple Model of galactic chemical evolution.

[4 marks]

e) In the Simple Model of galactic chemical evolution, the change δZ in the heavy element mass fraction Z is given by

$$\delta Z \;=\; -\,p\,\frac{\delta M_{gas}}{M_{gas}} \quad .$$

where p is a parameter called the yield.

Derive an expression for Z as a function of time t, and thence show that the mean metallicity of a population of long-lived stars is given by

$$\langle Z \rangle \;=\; p \left(\frac{M_{\mathsf{gas}}(0)}{M_{\mathsf{stars}}} - 1 \right) \ln \left(1 - \frac{M_{\mathsf{stars}}}{M_{\mathsf{gas}}(0)} \right) \;+\; p \;.$$

(You may quote the standard integral

$$\int \ln(1 - x/a)\, \mathrm{d}x \;=\; (x - a)\ln(1 - x/a) - x + \text{constant} \qquad).$$

[7 marks]

f) Describe the "G-dwarf problem", and discuss some possible modifications to the Simple Model which may help to resolve it.

[4 marks]

End of Paper