Queen Mary University of London

MTH5123 **Differential Equations,**

Formative Assessment Week 4 **G.Bianconi**

- This Formative Assessment consists of three parts:
  I. Practice problems. You will get help on this Formative Assessment in the tutorial of week 3. You should work on this before you go to this session.
  II. Mock Quiz Week 4.
  III. Exploration problems (to help you understand concepts discussed during lecture, not optional and examinable)
- A selection of solutions to the listed problems will be posted on QMPlus by the end of Week 4. You are expected to seek solutions to the remaining problems using the Reading List and making use of the tutorial sessions.
- I encourage all students to learn and check their computational answers using math softwares such as Mathematica, MATLAB, etc. Using numerical software is a fun practice and will help you to visualise your solutions.

I. Practice Problems

A. Find the general solutions of the following linear homogeneous differential equations of second order:

1) $y'' + y' - 12y = 0$

2) $6y'' + 5y' - 6y = 0$

3) $y'' + 2y' + 17y = 0$

4) $y'' + 2y' + 3y = 0$

5) $16y'' + 8y' + y = 0$

B. Solve the following initial value problems:

1) $10y'' - y' - 3y = 0, \quad y(0) = 1,\ y'(0) = 0$

2) $y'' - 2y' - 3y = 0, \quad y(0) = 2,\ y'(0) = -3$

3) $y'' - 4y' - 5y = 0, \quad y(0) = -1,\ y'(0) = -1$

4) $y'' - 4y' + 13y = 0, \quad y(0) = 4,\ y'(0) = 0$

C. Assign to each of the following linear homogeneous differential equations

1) $2y'' - 8y' + 8y = 0$ **2)** $y'' + y' - 2y = 0$ **3)** $y'' + 2y' + 2y = 0$

a correct solution from the list:

i) $y = e^{-x}\left(2\cos x - \sqrt{2}\sin x\right)$ **ii)** $y = e^{x} + \frac{1}{7}e^{-2x}$ **iii)** $y = e^{2x}(x+1)$.

D. Determine the general solution for the homogeneous linear differential equation

$$y'' - 2y' + y = 0.$$

Fix the constants of integration by the initial condition $y(2) = 1$, $y'(2) = -2$ and write down the explicit form of the corresponding solution to the initial value problem.

II. Mock Quiz Week 4

Check your understanding with Mock Quiz Week 4.

III. Further Exploration: More Practice with 2nd Order Linear ODEs

A. In each exercise below, solve the initial value problem and determine the value of α (if any) so that the solution approaches zero as $t \longrightarrow \infty$. Sketch/Graph the solution curve.

1) $\ddot{y} + 5\dot{y} + 6y = 0$, $y(0) = \alpha$, $\dot{y}(0) = 3$.

2) $4\ddot{y} - y = 0$, $y(0) = 2$, $\dot{y}(0) = \alpha$

3) $\ddot{y} + (2\alpha - 1)\dot{y} + \alpha(\alpha - 1)y = 0$

B. Consider the equation $a\ddot{y} + b\dot{y} + cy = f$, where a, b, c *and* f are all constants. Find all constant (equilibrium) solutions of this ODE. Let y_{eq} denote an equilibrium solution to the equation and set $Y = y - y_{eq}$, measuring the deviation of a solution y from an equilibrium solution. Find the differential equation satisfied by Y. Why (or in what circumstances) might we choose to study the differential equation for Y instead of the original equation?