MTH5123 **Differential Equations**

Formative Assessment Week 3 **G.Bianconi**

- This Formative Assessment consists of three parts:
  - **I.** Practice problems. You will get help on this Formative Assessment in the tutorial of week 3. You should work on this before you go to this session.
  - **II.** Mock Quiz Week 3.
- A selection of solutions to the listed problems will be posted on QMPlus by the end of Week 3. You are expected to seek solutions to the remaining problems using the Reading List and making use of the tutorial sessions.
- I encourage all students to learn and check their computational answers using math softwares such as Mathematica, MATLAB, etc. Using numerical software is a fun practice and will help you to visualise your solutions.

I. Practice Problems

A. Determine which of the following differential equations are exact differential equations. If an equation is exact, determine its general solution first in implicit form and then (if possible) in explicit form.

1) $(1 - y\sin(x)) + \cos(x)y' = 0$

2) $\frac{x}{\sqrt{x^2+y^2}} + \frac{y}{\sqrt{x^2+y^2}}\, y' = 0$

3) $-x + (x - y)y' = 0$

4) $x^2 + y/x + \ln|xy|\frac{dy}{dx} = 0$

5) $2xy^2 + 4x^3 + 2(x^2 + 1)yy' = 0$

B. When solving the following two exercises on exact equations, sharpen your writing skills by explaining your reasoning in complete sentences and justifying your steps.

1) Find a value of the parameter b such that the following differential equation is exact and solve it for that value of the parameter:

$$\frac{y - x\,b}{yx} + \frac{x}{y^2}y' = 0\,.$$

2) Find all functions $f(y)$ such that the following differential equation becomes exact:

$$x^2 + \frac{f(y)}{xy} + y'\ln|xy| = 0$$

and solve it in implicit form for a particular choice such that $f(1) = 1$.

C. Determine the general solution of the exact differential equation

$$1 - \frac{x}{x^2 + y^2} - \frac{y}{x^2 + y^2}y' = 0\,.$$

Write down the explicit solution for the initial condition $y(0) = e$.

D. Consider the initial value problem

$$\frac{dy}{dx} = f(x, y), \quad f(x, y) = \sqrt{y^2 + p^2}, \quad y(1) = 0\,,$$

where $p > 0$ is a real parameter. Show that the Picard-Lindelöf Theorem ensures existence and uniqueness of a solution to the above IVP on a rectangular domain $|x - 1| \leq A$, $|y| \leq B$. Find the value of the Lipschitz constant K for the above problem for a given A and B. Write down the maximal value of the width A for a given value of B.

II. Mock Quiz Week 3

Check your understanding with Mock Quiz Week 3.