**MTH5123** **Differential Equations**

**Formative Assessment _ Week 2** **G.Bianconi**

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- This Coursework consists of three parts:
  - **I.** Practice problems (you will get help on this part in session 4 of week 2. You should work on this before you go to this session.)
  - **II** Mock Quiz Week 2.
  - **III.** Exploration problems (to help you understand concepts discussed during lecture, not optional and examinable)
- A selection of solutions to coursework problems will be posted on QMPlus after the coursework deadline. You are expected to seek solutions to the remaining problems using the Reading List and making use of the tutorial sessions.
- I encourage all students to learn and check their computational answers using math softwares such as Mathematica, MATLAB, etc. Using numerical software is a fun practice and will help you to visualise your solutions.

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## I. Practice Problems

**A.** Determine the general solutions of the following differential equations. For each solution fix the arbitrary constant according to the given initial condition.

**1)** $y' = -xy, \quad y(0) = -2$

**2)** $y' = x\cos(x)y, \quad y(0) = 1$

**3)** $y' = -y/(1+x), \quad y(0) = -1$

**4)** $y' = y/(4-x^2), \quad y(0) = 1$

**5)** $y' = y/(x^2+2x+2), \quad y(0) = 2$

**B.** Solve the initial value problems associated with the following inhomogeneous linear differential equations.

**1)** $y' = y\frac{3x^2}{1+x^3} + x^2 + x^5, \quad x > -1, \quad y(0) = -1$

**2)** $y' = -y\tan x + \cos x, \quad -\pi/2 < x < \pi/2, \quad y(0) = 2$

**C.** Determine the general solution of the following differential equations

**1)** $y' = 3y + 5, \quad y(0) = -2$

**2)** $y' = -2xy + 2x, \quad y(0) = 0$

and solve the associated initial value problems.

**D.** Determine the general solution to the linear inhomogeneous differential equation

$$y' = \frac{x}{1+x^2}y + \sqrt{\frac{1+x^2}{1-x^2}}.$$

**E.** Determine the general solution of the following reducible to separable differential equation

$$y' = -y/x - x/y.$$

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## II. Mock Quiz Week 2

Check your understanding with Mock Quiz Week 2.

## III. Further Exploration: Integrating Factors

**A.** In the Week 2 Lecture Notes, there is a reference to solving first-order linear ODEs using the "*Integrating Factor Method* from Calculus 2." We shall learn (or review?!) this method in the subsequent exercises. Consider the differential equation

$$\frac{dy}{dx} + \frac{1}{2}y = \frac{1}{2}e^{x/3}.$$

**1)** Using techniques discussed in lecture, find the general solution to this equation and sketch the integral curve passing through the initial condition $y(0) = 1$.

**2)** In the next three exercises, we now use the integrating factor method to solve this differential equation a second time. Multiply the ODE by the function $\mu(x)$ and compare the left hand side with the quantity

$$\frac{d}{dx}\left[\mu(x)y\right] = \frac{d\mu}{dx}y + \mu\frac{dy}{dx}.$$

What differential equation must $\mu(x)$ satisfy in order for the left side of the original ODE to agree with the equation above? *Answer:* $\frac{d\mu}{dx} = \frac{1}{2}\mu$.

**3)** Complete the following sentence: A function whose derivative equals $\frac{1}{2}$ times the original function is given by [ ]. Your answer to this sentence can be checked using separation of variables or ordinary integration, depending on your approach.

**4)** Verify that by using $\mu(x) = Ce^{x/2}$, the original ODE can be rewritten as

$$\frac{d}{dx}\left[e^{x/2}y\right] = \frac{1}{2}e^{5x/6}.$$

Integrate both sides of this equation to find the general solution

$$y(x) = \frac{3}{5}e^{x/3} + Ce^{-x/2}.$$

Once you've imposed the initial condition $y(0) = 1$, compare your answer with the solution you found using the methods from lecture in the first exercise of this section.

**B.** *More challenging, but achievable:* Using the above example as a guide, can you write down a general procedure for using an integrating factors $\mu(x)$ to solve a general first-order linear ODE of the form

$$\frac{dy}{dx} = A(x)y + B(x)?$$