Practice questions

Cour Formative Assessment 1

A.1) $y' = 3x^2 + 2x + 3$ & $y(0) = 1$

$y(x_0) = y_0$

$x_0 = 0 \quad y_0 = 1$

$y' = f(x) = 3x^2 + 2x + 3$

$y(x) = \int f(x)\,dx + C$ General solution

$y(x) = \int (3x^2 + 2x + 3)\,dx + C$

$\boxed{y(x) = x^3 + x^2 + 3x + C}$ General solution

$1 = y(0) = C$ $\boxed{C = 1}$

$\boxed{y(x) = x^3 + x^2 + 3x + 1}$

(A.3) $y' = (x+1)\, e^{-x^2-2x}$ & $y(0) = 1$

$$y' = f(x) = (x+1)\, e^{-x^2-2x} \qquad y(x_0) = y_0$$

$$x_0 = 0 \qquad y_0 = 1$$

$$y(x) = \int f(x)\, dx + C$$

$$y(x) = \int \underbrace{(x+1)\, e^{\overbrace{-x^2-2x}^{w}}}_{I}\, dx + C$$

$$\boxed{w = -x^2 - 2x}$$

$$\frac{dw}{dx} = -2x - 2 = -2(x+1)$$

$$dw = -2(x+1)\, dx \quad \Rightarrow \quad (x+1)\, dx = -\frac{1}{2}\, dw$$

$$y(x) = -\int e^{w}\, \frac{dw}{2} + C = -\frac{1}{2}\int e^{w}\, dw + C = -\frac{1}{2} e^{w} + C$$

General solution

$$\boxed{y(x) = -\frac{1}{2}\, e^{-x^2-2x} + C}$$

$\Rightarrow$ I.C. $y(0) = 1$

$$1 = y(0) = -\frac{1}{2} e^{0} + C = -\frac{1}{2} + C \quad \Rightarrow \quad C = \frac{3}{2}$$

$$\boxed{y(x) = -\frac{1}{2}\, e^{-x^2-2x} + \frac{3}{2}}$$

Exponential growth / decay

$y' = ky$ & $y(0) = 2$

$y' = f(x)\,g(y)$ separable ODE

I.C.

$y(x_0) = y_0$

① $f(x) = k$, $g(y) = y$ $\qquad x_0 = 0,\ y_0 = 2$

② $\dfrac{dy}{dx} = ky \qquad \displaystyle\int \frac{dy}{y} = \int k\,dx + C'$

③ LHS: $H(y) = \displaystyle\int \frac{dy}{y} = \ln|y|$

RHS: $F(x) = \displaystyle\int k\,dx = kx$

$H(y) = F(x) + C'$ implicit solution

$\ln|y| = kx + C$

③ $H(y) = \ln|y| = u \qquad |y| = e^u$

$y = H^{-1}(u) \qquad y = \pm e^u$

$$u = F(x) + C' = \lambda x + C'$$

$$y(x) = \pm e^{\lambda x + C'} = \underbrace{\pm e^{C}}\, e^{\lambda x} = D\, e^{\lambda x}$$

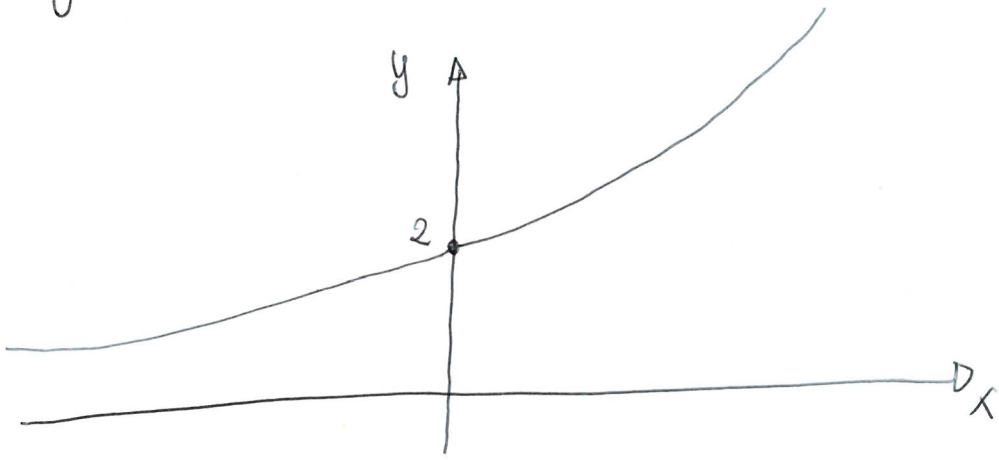
$$\boxed{y(x) = D e^{\lambda x}}$$ Explicit solution

I.C. $y(0) = 2$

$$2 = y(0) = D e^{\lambda \cdot 0} = D e^{0} = D \quad \Rightarrow D = 2$$

$$y(x) = 2 e^{\lambda x}$$

If $\lambda > 0$



If $\lambda < 0$