Simple Pendulum

**IMPORTANT:** You must work through the derivation required for this assignment before you turn up to the laboratory. You are also expected to know what the arithmetic average and standard deviation of an ensemble of data are. If you miss the relevant lecture you will need to read up on the background material in preparation.

**Aim**

You will probably have encountered a description of, or performed, a simple pendulum experiment before. Therefore, it is fair to ask yourself why are you encountering this elementary example of experimental physics at undergraduate level? The answer is simple, *we want you to think deeply about what you can do with the apparatus, and to do so while applying your knowledge of statistics*. Having done this we want you to reflect on how you can move toward a better, more refined, methodology and estimate what that might produce in terms of an improved result.

In essence, this is an experiment designed for you to use your knowledge of data analysis (the mean, standard deviation, error propagation) to arrive at a well-defined measurement of $g$.

The final point is as important as the prelude, to explore the validity of the method used. At what point will the simple pendulum approximation fail to be valid given the data you are collecting? Are there other limitations or prospects for this methodology, etc?

You are expected to have familiarised yourself with this script before you turn up to the lab. This will ensure that you will be able to complete the required work, and therefore maximise your chances of a good grade for this assignment.

**Background physics**

The value of acceleration due to gravity ($g$ typically reported in units of $\text{m/s}^2$) changes slightly depending on where one makes the measurement. Precise maps of $g$ are used in geological surveys as small local deviations in $g$ may be indicative of mineral reserves, such as oil or gas. There are several ways of measuring this quantity and the one discussed here is the use of a swinging (simple) pendulum. This can be described as a simple harmonic oscillator with mass $m$, suspended on a string of length $L$. The period of oscillation $T$ is given by $2\pi/\omega$, where $\omega$ is the angular frequency and is given by $\sqrt{g/L}$. The corresponding period of oscillation for the pendulum is given by,

$$T = 2\pi \sqrt{\frac{L}{g}} \quad (1.1)$$

This is valid for small oscillations, as the small angle approximation of $\sin \theta \approx \theta$ is used in deriving the relationship between $T$ and $g$. Using Eq. (1.1) it is possible to estimate the acceleration due to the Earth’s gravity from measurements of (i) the length, $L$, and (ii) the period of oscillation, $T$. There is no dependence on the amplitude of oscillation (provided the small angle approximation remains valid) or the mass of the bob at the end of the pendulum. Eq. (1.1) can be re-arranged as follows

$$g = \frac{4\pi^2 L}{T^2} \quad (1.2)$$

so that one can directly obtain $g$ from a single measurement or data point. This is the initial aim of the experiment. Following on from this we expect you to make use of your knowledge of statistics to develop refined measurements and explore systematic errors associated with this experiment.
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DERIVATION REQUIRED: Starting from the error propagation formula given in Eq. (1.3)

\[ \sigma_f^2 = \left( \frac{\partial f}{\partial x} \right)^2 \sigma_x^2 + \left( \frac{\partial f}{\partial y} \right)^2 \sigma_y^2 \]  

(1.3)

for two measurements \( x \) and \( y \), with uncertainties \( \sigma_x \) and \( \sigma_y \), respectively one can determine the variance \( \sigma_f^2 \) and hence uncertainty \( \sigma_f \) of some function of \( x \) and \( y \): \( f = f(x,y) \). Apply this equation to the problem at hand in order to determine the uncertainty on \( g \) in terms of the values and uncertainties of \( L \) and \( T \).

Experimental setup

The basic experimental setup is shown in Figure 1. One can see a retort stand holding a mass (the bob) that is suspended by a string. The mass has a small permanent magnet attached to its base. You will need to measure the length, \( L \), of the pendulum and determine the expected measurement uncertainty \( \delta L \) on this measurement. Carefully justify your choice for this uncertainty. You should assume that the bob can be approximated by a point mass but consider the effect of the other parts of the pendulum.

Important: We will distinguish between a measurement uncertainty and the statistical uncertainty. The former is the expected uncertainty on each measurement. It arises from the the calibration error of the instrument used. The latter is associated with the statistical spread of a large number of measurements and is usually calculated as the standard deviation of the measurements. We will denote the former using the symbol \( \delta x \) and the latter using the symbol \( \sigma \). See the Course Notes for more on these kinds of uncertainties.

Stopwatch timing

One can use a stopwatch to determine the period for one full oscillation \( T \). What do you think the measurement uncertainty \( \delta T \) is on any one measurement of \( T \)? Is it related to your reaction time? Discuss this issue.

MEASUREMENT (1): Record the period \( T \) of a single oscillation of the pendulum using a stop clock. Use this single measurement, the measurement uncertainty \( \delta T \), and your measurement of \( L \pm \delta L \) to determine \( g \pm \delta g \).

MEASUREMENT (2): Now record 10 measurements for a single oscillation of the pendulum using a stop clock. From these measurements you can calculate a standard deviation which will be your measure of the statistical uncertainty in \( T \). Compare this statistical uncertainty with the measurement uncertainty you recorded earlier. Are they comparable? Discuss. Use the statistical uncertainty in \( T \) to determine \( g \pm \sigma_g \). Is this a better measurement? Discuss.

MEASUREMENT (3): Now record 10 measurements for 10 oscillations of the pendulum using a stop clock. What is the uncertainty on \( T \) now? Use these data to calculate \( g \pm \sigma_g \) and compare this value with your previous calculations. Discuss.
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Arduino timing

[The Arduino programme for this experiment is provided at the end of the assignment (for reference) and is available online (See the QM+ page). You should cut and paste the on-line version into the Arduino to save time and avoid transcription errors. You must read the on-line wiki for information on using the Arduino before coming to the laboratory.]

Timing using a stopwatch is not optimal as the uncertainty in timing one period of oscillation is relatively large. As a result, one can implement an automated data acquisition (DAQ) system using an Arduino connected to a hall probe. The magnet on the base of the pendulum bob changes the voltage from the hall probe, which can be used to start and stop a clock running at one kHz in the Arduino. This gives a resolution on timing at the level of a millisecond. However, one should still consider systematic uncertainty on timing arising from the experimental setup. The circuit for the Arduino DAQ system is shown in Figure 2. The pins on the Hall probe need to be connected as follows:

- Pin 1: +5 V
- Pin 2: 0 V (GND)
- Pin 3: Analogue output (A0)

The Hall probe will need to be placed so that the magnet passes close by when the pendulum swings past it. As a rule of thumb, keep the separation of around 1 mm between the two.

The number of oscillations that are timed by the system is specified by the following line of code:

\[
\text{int} \text{periods} = 1; \quad (1.4)
\]

You can change the number of periods from 1 to a higher number to time for more than one period of oscillation.

MEASUREMENT (4): Set the Arduino to time a single period. Compile and upload the code. Using this DAQ system record 10 measurements of a single period of oscillation. Calculate the statistical error on this set of measurements and \( g \pm \sigma_g \). Compare with the earlier measurements and discuss.
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Figure 2: The Arduino circuit diagram to be used for the simple pendulum experiment. 5V, GND and A0 should be connected to pins 1, 2 and 3 of the Hall sensor respectively. The schematic of the Hall sensor shows which pin is which.

**MEASUREMENT (5):** Set the Arduino to time 10 periods. Compile and upload the code. Using this DAQ system record 10 measurements of 10 periods of oscillation. Calculate the statistical error on this set of measurements and evaluate $g \pm \sigma_g$. Compare with the earlier measurements and discuss.
Systematic uncertainties

So far, we have assumed that the uncertainties in $T$ arise from random errors. But what about systematic errors? List possible sources of systematic errors in this experiment (3 will do).

But what about the DAQ system? Here the Arduino timer is quite accurate. So where do the uncertainties come from? In fact, are the errors now random, or are they systematic?

**MEASUREMENT (6):** Set the Arduino DAQ to record 10 periods of oscillation. Start the oscillations with a relatively large amplitude (of say 10 cm). You will need to ensure that the pendulum does not precess significantly. Now record the period of the pendulum till the amplitude decreases to about 1 cm. This will take time, but once this is set up correctly and you are sure that the periods are being recorded correctly (i.e., no precession), you may leave the setup alone and carry on with your calculations from the earlier parts of the experiment. Once done, plot the period versus measurement number. Since the measurement number is roughly proportional to time, you have, in effect, plotted period versus time. You may ignore error bars on this plot.

What do you observe? Are the readings of successive period measurements correlated or uncorrelated? Does the period change randomly or systematically? What are the consequences of this for the observations made with the Arduino DAQ? Can you regard those ten successive measurements of the period as independent, randomly distributed measurements? If not, why not? If yes, why?

Consider these and any other questions you may yourself ask in the analysis/ reflection part of the writeup.

Reflection

An important part of any experimental investigation is to step back and think about what you have done, what it means, and what could be improved next time. Reflect on your work with the simple pendulum, in particular, on how you have used statistics to refine your understanding of the problem and any limitations of the experimental setup you have identified. Note down any possible improvements to the methodology that could be used in the future, and where appropriate quantify the effect of said improvements.

Writing your report

The report must include:

1. A title.
2. Author’s name and affiliation.
3. An abstract, which is a summary of a few lines including results but not too detailed.
4. A short introduction — what are you describing, and why did you do it?
5. A brief summary of the theory. Do not include theoretical details that are not relevant to your experiment, especially if you do not fully understand them.
6. A brief description of your experimental set-up.
7. A brief description of what you did and measured.
8. A summary of the results, together with calculated quantities. Make sure you present ALL the important measurements needed for the final results including the errors, which a discussion of how you determined those errors.
9. Include a table of the key results (e.g. $T$ and $g$ along with their errors).
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10. Show raw data as graphs (much more informative than tables which you will not have the space to include) where appropriate.
11. A discussion of the significance of the results, including any uncertainties due to measurement precision (errors) and whether any differences found are meaningful.
13. A list of references with name(s) of author(s), journal name and volume number or book title and publisher, page number(s), and date.

Submission on QM+

Your first formal report will be based on this experiment. Details of this are provided on QM+

Create this document using MS Word and save it as a PDF file. We will not accept submissions in any other format. The submission deadline will be posted on QM+

Arduino code

The Arduino code is written in C. Those who are interested in developing a more detailed understanding of the C programming language might consider looking in the Library for the classic reference used by generations of programmers: the book by Brian Kernighan and Dennis Ritchie, “The C Programming Language”.

/*
Hall sensor timing code
Navot Arad, Queen Mary University of London
Modified: AIM 21 Sep 2015
*   periods now does refer to a period!
*   Message printed out stating number of periods.
*/

// number of oscillations measured
int periods = 1;
int swings = 0;
int pin = 0, hall = 0, time = 0, start = 0;
int calibration = 510, count = 0, swing = 0;
int uplim = 0, lowlim = 0;
float seconds = 0;

void setup() {  
    Serial.begin(115200);
    Serial.print("Starting timing for ");
    Serial.println(periods);
    delay(1000);
    swings = 4 * (periods);
    uplim = calibration + 30;
    lowlim = calibration - 30;
    cli(); //stop interrupts
    // set timer1 interrupt at 1kHz
    TCCR1A = 0; // set entire TCCR1A register to 0
    TCCR1B = 0; // same for TCCR1B
    TCNT1 = 0; // initialize counter value to 0
    // set timer count for 1kHz increments
    OCR1A = 1999; // = (16*10^6) / (1000*8) - 1
    // had to use 16 bit timer1 for this bc 1999>255, but could switch to timers 0 or 2 with larger prescaler
    // turn on CTC mode
    TCCR1B |= (1 << WGM12);
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// Set CS11 bit for 8 prescaler
TCCR1B |= (1 << CS11);  
// enable timer compare interrupt
TIMSK1 |= (1 << OCIE1A);
sei(); //enable interrupts

// Interrupt function, runs at 1kHz
ISR(TIMER1_COMPA_vect){
    hall = analogRead(pin);
    //Serial.println(hall);
    if (count > 0){
        start = 1;
    }
    if (start == 1){
        time ++;
    }
    if (hall > uplim or hall < lowlim){
        swing ++;
    }
    if (hall > lowlim and hall < uplim and swing != 0){
        count ++;
        swing = 0;
    }
}

void loop() {
    if (count == (swings + 1)){
        seconds = time / 1000.0;
        Serial.println(seconds, 3);
        //Serial.print("Count = ");
        //Serial.println(count);
        delay(100);
        time = 0;
        start = 0;
        count = 0;
    }
}