Queen Mary
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MTH5123 Differential Equations

Formative Assessment: Week 11

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- Each Coursework consists of two parts:
  - **I.** Practice problems
  - **II.** Mock Quiz
- A selection of solutions to the listed problems will be posted on QMPlus by the end of Week 11 and discussed during the tutorials.
- I encourage all students to learn and check their computational answers using math software such as Mathematica, MATLAB, etc. Using numerical software is a fun practice and will help you to visualise your solutions.

I. Practice Problems

A. Find all fixed points of the system $\dot{y_1} = f_1(y_1, y_2)$, $\dot{y} = f_2(y_1, y_2)$ with

$$f_1(y_1, y_2) = 2y_1 + y_2^2 - 1 \ , \ f_2(y_1, y_2) = 6y_1 - y_2^2 + 1 \ .$$

Investigate the linear stability of the corresponding system linearized around each fixed point. Describe the type of fixed point and explain in words the shape of the trajectories close to it.

B. Find the values of the real parameter a for which the system (This content might show up in week 12 session 1 if not in week 11 session 3)

$$\dot{y_1} = y_2 + ay_1 - y_1^5 \ , \ \dot{y} = -y_1 - y_2^5$$

has a stable fixed point at $y_1 = y_2 = 0$. Use the function $V(y_1, y_2) = y_1^2 + y_2^2$ for judging the stability of the nonlinear system when linear stability analysis is insufficient.

C. Study the stability properties of the solution $y_1(t) = 0$ of the second order differential equation

$$\ddot{y_1} + (a-1)\dot{y_1} + (4-a^2)y_1 = 0$$

for the values $a = 1$, $a = 2$ and $a = -2$ of the real parameter a by converting the ODE to a system of two first-order equations (using week 3 knowledge).

D. Recall from Coursework 9, the following differential equation modelling the flow of electric current in a simple series circuit:

$$L\frac{d^2I}{dt^2} + R\frac{dI}{dt} + \frac{1}{C}I = 0,$$

where L, R and C are positive constants. Rewrite the equation as a system of two first-order differential equations for $y_1 = I$ and $y_2 = dI/dt$. Show that $y_1 = 0$, $y_2 = 0$ is a critical point of the system and analyze the nature and stability of the critical point as a function of the parameters L, R and C.

Remark: Observe that your analysis would apply equally well to the equation of motion for a damped spring-mass system

$$m\frac{d^2u}{dt^2} + c\frac{du}{dt} + ku = 0,$$

introduced in the first week of lectures!

II. Mock Quiz

Train yourself for Coursework 2 by answering Mock Quiz Week 11.