

This sheet contains questions for you to work through in your tutorial, singly or in a group.

It's important to work through lots of questions for practice. Remember that mathematics is not a spectator sport! If you want more questions, look at the "Extra questions" sheets on QMPlus.

1 As before, this question is to encourage you to try filling in a piece of a proof skipped in lecture.

Let R be a ring. Prove the left distributive law for $M_2(R)$.

2 Let R be a ring. Prove that the set

$$\left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} : a, b, c \in R \right\}$$

of 2×2 matrices over R whose lower left entry equals zero is a ring, with the usual addition and multiplication of matrices. You may assume that $M_2(R)$ is a ring.

3 Label the corners of a regular pentagon 1 through 5 in order. Each of the symmetries of the pentagon—reflections, rotations, and the identity (doing nothing)—gives a permutation in S_5 describing how it moves the corners.

(a) Write down the set of all these permutations.

(b) Is your set closed under composition?

4 A *permutation matrix* in $M_n(\mathbb{R})$ is a matrix so that, in each row and column, there is exactly one 1, and all other entries are 0. For example, an identity matrix of any size is a permutation matrix.

(a) Pick some $n \geq 3$ and write down two more permutation matrices in $M_n(\mathbb{R})$, other than the identity matrix. Then compute their product.

(b) Why is the name "permutation matrix" appropriate?