

You are to write up a careful and professionally presented solution to the question below. This is to be submitted on QMPlus as a single PDF or JPEG file by 12:00 noon, Monday 4 April 2022.

To submit For a natural number n , let

$$C_n = \{(a_{ij}) \in M_n(\mathbb{R}) : a_{ij} = a_{kl} \text{ for all } i, j, k, l \text{ such that } j - i \equiv_n l - k\}.$$

- (a) Exactly one of the two matrices below is an element of C_3 . Which one? [1 mark]

$$A_1 = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 0 \\ 2 & 0 & 1 \end{pmatrix} \quad A_2 = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{pmatrix}$$

- (b) Let A be the matrix you chose in part (a). Compute A^2 . Check that $A^2 \in C_3$, and write out a brief explanation (one or two sentences is enough) explaining how you checked. [2 marks]
- (c) Give a complete proof that C_3 is a ring. You may assume that $M_3(\mathbb{R})$ is a ring. Hint: make as much use of that assumption as possible in your proof. [7 marks]