MTH4104 **Introduction to Algebra**

Week 8 tutorial questions **w/c 14 March 2022**

This sheet contains questions for you to work through in your tutorial, singly or in a group.

It's important to work through lots of questions for practice. Remember that mathematics is not a spectator sport! If you want more questions, look at the "Extra questions" sheets on QMPlus.

1 Let $f = [2]_8 x + [3]_8$ and $g = [4]_8 x^2 + [6]_8 x + [3]_8$ be elements of $\mathbb{Z}_8[x]$. Compute the sum $f+g$ and product fg.

2 Let R be a skewfield. Let f and g be nonzero polynomials in $R[x]$, of degrees m and n, respectively.

(a) Is $\deg(fg)$ uniquely determined by this information? If so, what is it? If not, what are the possible values it can take?

(b) The same questions for $\deg(f+g)$.

3 In lectures I didn't prove that $R[x]$ was a ring. This question is to get you to try filling in a piece of that proof.

Let R be a ring. Prove the left distributive law for $R[x]$.

4 Recall from section 4.1 of the notes that, for any ring R, each polynomial $f \in R[x]$ determines a function $R \to R$, which we usually also call f.

Give an example of a non-commutative ring R, two polynomials $f, g \in R[x]$, and an element $r \in R$ such that $(fg)(r) \neq f(r) \cdot g(r)$.

[To answer this question, of course, you must know an example of a non-commutative ring. If you don't yet know any, come back to this question in a fortnight, after we have studied matrices.]

5 Give an example of a finite ring R and a function $f : R \to R$ that is not a polynomial function, in the sense of section 4.1 of the notes. Justify your answer.