Queen Mary
University of London

MTH4104 **Introduction to Algebra**

Week 8 coursework **due 21 March 2022**

You are to write up a careful and professionally presented solution to the question below. This is to be submitted on QMPlus as a single PDF or JPEG file by 12:00 noon, Monday 21 March 2022.

To submit Let $\mathbb{R}[[x]]$ be the set of all expressions

$$a = a_0 + a_1x + a_2x^2 + \cdots = \sum_{i=0}^{\infty} a_i x^i$$

where $a_i \in \mathbb{R}$ for all nonnegative integers i. Informally, an element of $\mathbb{R}[[x]]$ is like a polynomial except that it can have infinitely many terms.

(a) Carefully write down definitions of addition and multiplication operations for $\mathbb{R}[[x]]$, analogous to the definitions for $\mathbb{R}[x]$ in the notes. Given $a, b \in \mathbb{R}[[x]]$, your definitions should indicate what each coefficient of the sum $a+b$ and product ab is. [2 marks]

(b) Let $f = a_0 + a_1x + \cdots + a_nx^n$ be a polynomial. I can treat f as an element of $\mathbb{R}[[x]]$ by defining $a_{n+1}, a_{n+2}, \ldots$ all to equal 0. This shows that $\mathbb{R}[x] \subseteq \mathbb{R}[[x]]$.

If you had already proved that $\mathbb{R}[[x]]$ was a ring, how could you use this fact to help you prove $\mathbb{R}[x]$ is a ring? [2 marks]

(c) Let $a \in \mathbb{R}[[x]]$ with $a_0 \neq 0$. Prove that a has a multiplicative inverse in $\mathbb{R}[[x]]$. You may assume that the multiplicative identity element in $\mathbb{R}[[x]]$ is

$$1_{\mathbb{R}[[x]]} = 1 + 0x + 0x^2 + 0x^3 + \cdots,$$

and that multiplication in $\mathbb{R}[[x]]$ is commutative. [6 marks]

[Hint. If $ab = 1_{\mathbb{R}[[x]]}$, equate coefficients and solve for $b_0, b_1, b_2, \ldots$ in turn.]