

This sheet contains questions for you to work through in your tutorial, singly or in a group.

It's important to work through lots of questions for practice. Remember that mathematics is not a spectator sport! If you want more questions, look at the "Extra questions" sheets on QMPlus.

- 1** Give an example of a ring R and an element $a \in R$ such that $a \neq 0$ but $a^2 = 0$ in R .
- 2** Let R be the subset $\{[2a]_6 : a \in \mathbb{Z}\} = \{[0]_6, [2]_6, [4]_6\}$ of \mathbb{Z}_6 . Endow R with the same definitions of addition and multiplication used for \mathbb{Z}_6 .
- (a) Does R satisfy the identity law for multiplication? Justify your answer.
- (b) Is R a ring? Is R a field?
- 3** Let m be a positive integer.
- (a) Write up a careful proof of the associative law for addition for \mathbb{Z}_m .
- (b) Your proof in part (a) probably comes down to the associative law for addition for \mathbb{Z} , together with "taking care of the brackets" $[\cdot \cdot]_m$.
- Of all the axioms for a commutative ring with identity, which of them can be proved for \mathbb{Z}_m in a similar way?

- 4** Let R be a ring. Prove carefully that

$$(-a)b = -(ab) = a(-b)$$

for any elements $a, b \in R$. Name the ring axiom or proposition you are using at each step of your argument.