

MTH4104

Introduction to Algebra

Week 5 coursework

due 28 February 2022

You are to write up a careful and professionally presented solution to the question below. This is to be submitted on QMPlus as a single PDF or JPEG file by 12:00 noon, Monday 28 February 2022.

To submit Let S be the set

$$S = \{a + bk : a, b \in \mathbb{R}\},$$

where k is a formal symbol. Define addition and multiplication operations on S as follows: given elements $x = a + bk$ and $y = c + dk$ in S ,

$$x + y := (a + c) + (b + d)k,$$

$$xy := (ac + bd) + (ad + bc)k.$$

- (a) Prove both identity laws for S . Include a short explanation (one sentence is fine) of how you know what the identity elements are. (5 marks)
- (b) Prove that the multiplicative inverse law is false for S . [That is, don't just write down a counterexample; also prove that your counterexample is valid.] (5 marks)