Queen Mary University of London

MTH4104 Introduction to Algebra

Week 4 tutorial questions w/c 14 February 2022

This sheet contains questions for you to work through in your tutorial, singly or in a group.

It's important to work through lots of questions for practice. Remember that mathematics is not a spectator sport! If you want more questions, look at the "Extra questions" sheets on QMPlus.

1 (a) The greatest common divisor of 61 and 18 is 1. Using the extended Euclidean algorithm, find a pair of integers (x,y) such that $61x+18y=1$.

(b) Find *all* pairs of integers (x,y) such that $61x+18y=0$.

[Hint: your previous experience with solving equations may be more useful than the Euclidean algorithm here.]

(c) Find all pairs of integers (x,y) such that $61x+18y=1$.

[Hint: combine the previous two parts.]

2 If you use the extended Euclidean algorithm with inputs 272 and 200, you will find that $\gcd(272,200)=8$, and that $272\cdot(-11)+200\cdot 15=8$. You are welcome to do the computation yourself and double-check this, but for this question we will take it for granted.

(a) Find integers x and y such that $272x+200y=16$.

[Try to do this just using what I've told you, without running the algorithm.]

(b) Prove that there are no integers x and y such that $272x+200y=4$.

3 Which elements $x\in\mathbb{Z}_{17}$ satisfy the equation

$$[9]_{17}x+[1]_{17}=[11]_{17}\cdot[9]_{17}^{-1}\,?$$

Justify your answer.

4 How many of the elements of $\mathbb{Z}_{19}$ have multiplicative inverses? What about $\mathbb{Z}_{20}$? What about $\mathbb{Z}_{66}$?

Is there a way to calculate how many elements of $\mathbb{Z}_m$ have multiplicative inverses, without having to list and count them all? Hint: think about one prime factor of m at a time and which elements this factor "rules out".