

*You are to write up a careful and professionally presented solution to the question below. This is to be submitted on QMPlus as a single PDF file by 12:00 noon, Monday 14 February 2022.*

- To submit** (a) We have defined the relations  $\equiv_6$  and  $\equiv_{10}$  to be certain subsets of  $\mathbb{Z}^2$ . Their intersection

$$R = (\equiv_6) \cap (\equiv_{10})$$

is another subset of  $\mathbb{Z}^2$ , so it is also a relation on  $\mathbb{Z}$ . Complete the sentence: if  $a, b \in \mathbb{Z}$ , then  $aRb$  is true if and only if \_\_\_\_\_ . [2 marks]

[You do not have to “simplify” your answer. This part of the question is only about relations as sets vs. true-or-false statements.]

- (b) The relation  $R$  from part (a) is equal to  $\equiv_m$  for some natural number  $m$ . What is  $m$ ? [1 mark]
- (c) Based on your answer to part (b), complete the following statement with a formula for  $p$  in terms of  $m$  and  $n$ , **and** write down a proof of your statement. [7 marks]

Let  $m$  and  $n$  be positive integers. Then  $(\equiv_m) \cap (\equiv_n) = (\equiv_p)$ , where  $p =$  \_\_\_\_\_ .