Queen Mary University of London

**MTH4104** **Introduction to Algebra**

**Week 2 tutorial questions** **w/c 31 January 2022**

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*This sheet contains questions for you to work through in your tutorial, singly or in a group.*

*It's important to work through lots of questions for practice. Remember that mathematics is not a spectator sport! If you want more questions, look at the "Extra questions" sheets on QMPlus.*

**1** Let $R$ be the relation on the set of positive integers such that $R$ is true of two positive integers if and only if there is no multiple of $\pi$ between them.

(a) Write down $R$ in symbols as a subset of $(\mathbb{Z}_{>0})^2$. [Be careful with "between": have you covered all cases?]

(b) Prove that $R$ is an equivalence relation. You may assume that $\pi$ is irrational.

(c) Write down the equivalence class of 24 in $R$. You may use a calculator.

**2** If $R$ is a relation on a set $X$, then you need to state what $X$ is to fully specify the relation (just like you need to state the domain and codomain to fully specify a function). This question is about why $X$ is needed.

(a) Give an example of a set $R$ of ordered pairs and two different sets $X$ and $Y$ such that $R$ is a relation on $X$ and also a relation on $Y$.

(b) If $X$ and $Y$ are different sets, is it possible for $R$ to be an equivalence relation on $X$ and also an equivalence relation on $Y$? Justify your answer.

**3** Let $X = \{2,4,6,8,10\}$ and let the partition $\{A_1, A_2, A_3\}$ of $X$ be defined by

$$A_1 = \{2,8\}, \quad A_2 = \{4,6\}, \quad A_3 = \{10\}.$$

Write down the equivalence relation $R \subseteq X \times X$ determined by this partition.

**4** If $r$ and $s$ are real numbers, $\max(r,s)$ is defined to be whichever of $r$ and $s$ is greater. For example, $\max(5,3) = 5$.

Let $R$ be the following relation on the set $\mathbb{R}^2$. For two points $p,q \in \mathbb{R}^2$, let $p = (x_1,y_1)$ and $q = (x_2,y_2)$. Then $(p,q) \in R$ if and only if $\max(|x_1|,|y_1|) = \max(|x_2|,|y_2|)$. You may assume $R$ is an equivalence relation [i.e. you need not write out a proof of this].

State the partition associated to $R$ in words. Give the nicest, least "robotic" description you can.

**5** The following is a false proposition; it is missing a necessary assumption.

> **Proposition.** Let $X$ be any set and $Y$ any subset of $X$. Then $\{Y, X \setminus Y\}$ is a partition of $X$.

Set the proposition right by including the necessary assumption, and then prove it.