So far:

\[ 0 \text{ is } \frac{\mu-1}{\mu} \text{ attracting} \]

\[ 1 \text{ is attracting} \]

\[ 2 \text{ is attracting} \]

\[ 3 \]

\[ 4 \mu \]

For \( \mu > 3 \), both fixed points are repelling, but what is happening dynamically?

We will see that \( \mu = 3 \) marks the 'birth' of a period-2 orbit, and for those \( \mu > 3 \) which are "not too much larger than 3" this period-2 orbit is attracting.
Period-2 points:
These are solutions to \( f_\mu^2(x) = x \).

\[
f_\mu^2(x) - x = f_\mu(f_\mu(x)) - x = f_\mu(\mu x(1-x)) - x = \mu \cdot \mu x(1-x)(1-\mu x(1-x)) - x = -x + (\mu^2 x - \mu^2 x^2)(1-\mu x + \mu x^2)
\]

Check this as an exercise.

\[
f(x) - x = x (-\mu x + \mu - 1)(\mu^2 x^2 - (\mu^2 + \mu)x + (\mu + 1))
\]

The points of prime period 2 are the solutions of this quadratic, i.e., the two points \( p^\pm \) of prime period 2 are

\[
\frac{1}{2 \mu^2} \left( \mu^2 + \mu \pm \sqrt{(\mu^2 + \mu)^2 - 4 \mu^2 (\mu + 1)} \right)
\]
\[ p_{\pm} = \frac{1}{2\mu} \left( \mu + 1 \pm \sqrt{ (\mu+1)(\mu-3) } \right) \]

These notice that if \( \mu = 3 \) then both \( p_{+} \) and \( p_{-} \) are equal to the fixed point \( \frac{2}{3} = \frac{\mu-1}{\mu} \).

If \( \mu < 3 \) then \( p_{\pm} \) do not exist (we do not consider non-real solutions).

Is the 2-cycle \( \{ p_{+}, p_{-} \} \) attracting? Repelling?

Let \( M = \left| (f_{\mu}^2)'(p_{+}) \right| = \left| f_{\mu}'(f_{\mu}(p_{+})).f_{\mu}'(p_{+}) \right| \)

\[ = \left| f_{\mu}'(p_{-}).f_{\mu}'(p_{+}) \right| \]

\[ = \left| (\mu - 2\mu p_{-}) . (\mu - 2\mu p_{+}) \right| \]

\[ = \left| \mu^2 - 2\mu^2(p_{-} + p_{+}) + 4\mu^2 p_{-} p_{+} \right| \]
\[ \frac{e}{4} \begin{vmatrix} \mu^2 - 2\mu(\mu+1) + 4(\mu+1) \\ \mu + 2 \mu - \mu^2 \end{vmatrix} \]

Recall that the period-2 orbit is attracting if \( M < 1 \), and repelling if \( M > 1 \).

Now \( M = 1 \) when

- \( 4 + 2\mu - \mu^2 = 1 \)
- \( 4 + 2\mu - \mu^2 = -1 \)

\[4 + 2\mu - \mu^2 = 1 \iff \mu^2 - 2\mu - 3 = 0 \iff (\mu+1)(\mu-3) = 0\]

So \( M = 1 \) when \( \mu = 3 \) (we ignore the case \( \mu = -1 \) since we are only interested in values of \( \mu \) in the range \([0, 4]\))
\[ 4 + 2\mu - \mu^2 = -1 \]
\[ \iff \mu^2 - 2\mu - 5 = 0 \]

i.e. \[ \mu = \frac{1}{2} \left( 2 \pm \sqrt{4 + 20} \right) \]
\[ = \frac{1}{2} \left( 2 \pm 2\sqrt{6} \right) \]
\[ = 1 \pm \sqrt{6} \]

So \( M = 1 \) when \( \mu = 1 + \sqrt{6} \approx 3.449 \ldots \)

(we ignore the value \( 1 - \sqrt{6} \) since it is negative)

So \( M < 1 \) for \( 3 < \mu < 1 + \sqrt{6} \)

i.e. The 2-cycle \( \{ p-, p+ \} \) is attracting if \( 3 < \mu < 1 + \sqrt{6} \), and it is repelling if \( \mu > 1 + \sqrt{6} \).