

Main Examination period 2022 – May/June – Semester B

## MTH4000: Module Name

Examiners: F. Examiner, S. Examiner

This is "basic mode". This provides the basic formatting from the exam style file, but without the title page.

### Question 2 [25 marks].

- (a) Define what it means for a function to be **analytic** on some open subset of  $\mathbb{C}$ . [5]

Let  $s$  be a complex number such that  $\Re(s) > 1$ . Define  $\zeta(s)$  to be the number:

$$\zeta(s) := \sum_{n=1}^{\infty} n^{-s}.$$

(In all cases,  $n^{-s}$  means  $\exp(-s \log n)$ , with  $\log n \in \mathbb{R}$ .)

- (b) State how to define the analytic continuation of  $\zeta$  to all of  $\mathbb{C} \setminus \{1\}$ . [10]
- (c) Show that all non-real zeros of this analytic continuation have real part equal to  $\frac{1}{2}$ . [10]