

MTH9000 / MTH9000P: Sample Module

Examiners: F. Examiner, S. Examiner

This is optional. Here, one should put any special instructions (if any) relating to the exam. Otherwise, do not use this functionality.
For example: In this exam $0 \in \mathbb{N}$. Also, all expressions should be simplified as much as possible.

Question 2 [25 marks].

- (a) Define what it means for a function to be **analytic** on some open subset of \mathbb{C} . [5]

Let s be a complex number such that $\Re(s) > 1$. Define $\zeta(s)$ to be the number:

$$\zeta(s) := \sum_{n=1}^{\infty} n^{-s}.$$

(In all cases, n^{-s} means $\exp(-s \log n)$, with $\log n \in \mathbb{R}$.)

- (b) State how to define the analytic continuation of ζ to all of $\mathbb{C} \setminus \{1\}$. [10]
- (c) Show that all non-real zeros of this analytic continuation have real part equal to $\frac{1}{2}$. [10]