Vibrating plates and magnetic fluxes

A quick search on youtube for 'Chladni plate' will return numerous wonderful videos displaying the beautiful images that are created when sand is poured over a vibrating metal plate. By tuning the frequency, one can excite the various resonances k_n of the plate and 'see' how it vibrates - a technique first demonstrated by Chladni using a violin bow. Mathematically, this corresponds to finding the solutions to the Helmholtz equation

$$(-i\nabla)^2 \psi(x,y) = -\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2}\right)\psi(x,y) = k^2\psi(x,y),$$

subject to some boundary condition. Here $\psi(x,y)$ describes the amplitude of oscillation at the point (x,y).

In quantum mechanics, the same equation is used to describe the probability $|\psi(x,y)|^2$ of that a single particle, such as an electron, is likely to be at (x,y). One may then also consider the effects when a so-called 'Aharonov-Bohm flux-line' is threaded through the system - akin to the electron moving in the presence of an infinitely long wire with a certain current running through. The Helmoltz equation must then be modified to obtain

$$(-i\nabla + q\mathbf{A})^2 \psi(x,y) = k^2\psi(x,y),$$

with $\psi(x,y)$ now complex valued and $\mathbf{A}$ a suitable vector potential.

The project will be to first understand and compute the solutions to the Helmholtz equation using finite difference methods (a basic understanding in Matlab or other similar programming language is therefore required). This will be done for various shapes of plates. Afterwards the aim is to apply the same methods to the Aharanov-Bohm modified equation with one or more flux lines.

References

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[3] S. Rahav and S. Fishman, 'Spectral statistics of the rectangular billiard with a flux line', Foundations of Physics 31 (2001) 115.