SPA 3609 Tutorial 3, Questions for formative feedback

1. Find the *minimum energy* that an electron must have to produce Cherenkov radiation in silica (SiO_2, quartz) at a wavelength of 400 nm (remember that refractive index varies with wavelength).

   Calculate the minimum energy for a gamma-ray photon to produce (by Compton scattering) the energy of the electron you have just calculated in the first part of Q1.

   (a). We use the energy threshold relationship for Cherenkov radiation:

   $$\textbf{Energy} = m_e c^2 \left(\sqrt{1 + \frac{1}{n^2 - 1}} - 1 \right) \qquad \text{where we substitute } n = 1.47$$

   Energy = 0.186 MeV

   (b). For this amount of energy to be given to a Compton electron, the minimum gamma ray energy corresponds to a 180° scattering angle, with $E_e = (2\,E_\gamma^{\,2})/(m_e * c^2 + 2 * E_\gamma)$. Solving this equation for E_γ and substituting E_e= 0.186173 MeV gives us:

   $\mathbf{E_\gamma = -0.144}$ **MeV** **and** $\mathbf{E_\gamma = 0.330}$ **MeV**

   Clearly, the negative energy is not physical, so our solution is 330 keV.

2. Calculate the number of Cherenkov photons emitted per metre in water ice (take n = 1.3 at λ = 550 nm) if the photodetector has a uniform 50% response in the range 400 nm to 700 nm but completely insensitive outside the visible light range. Ignore the variation of refractive index with wavelength.

   $$\frac{dN_\gamma}{dx} = \int_{\lambda_1}^{\lambda_2} d\lambda \frac{d^2 N_\gamma}{dx d\lambda} = 2\pi z^2 \alpha \sin^2 \Theta_C \int_{\lambda_1}^{\lambda_2} \frac{d\lambda}{\lambda^2} = 2\pi z^2 \alpha \sin^2 \Theta_C \left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right)$$

   Cone opening angle is given by:

   $$\cos(\theta_c) = \frac{1}{n\beta}$$

   Assume beta ~ 1.0 to determine an angle of ~ 40° and assume single charge (z = 1). $\alpha = 7.3\times10^{-3}$ and finally you get 19700 photons per metre emitted (approximately). Putting in the detector quantum efficiency this results in 10000 detected Cherenkov photons per metre.

3. The semiconductor pn junction plays a very important role in modern tracking detectors as well as photodetectors.

   Intrinsic silicon is doped **n** type such that n = 10^{15} cm^{-3}, in this doped silicon calculate the thermal equilibrium concentration of holes.

Sketch the space-charge, internal electric potential and the internal electric field for an abrupt junction device.

$np = n_i^2$ where n_i is ~ 10^{10} cm^{-3} then p is 10^5 cm^{-3}

The diagram is in the lecture notes!

4. Explain qualitatively why the energy resolution of a silicon “ionisation” detector will be better that that of an air ionisation detector **for the same deposited energy** (say from the passage of a 100 MeV proton). Estimate by what factor it will be improved (assume resolution is determined by Poisson statistics).

Approximately 30 eV deposited in a gas will produce one ion-electron pair but about 3.5 eV will produce an electron-hole pair in silicon. Thus of order 10 times as many charge carriers will be produced for the same deposited energy. However given the effect of counting statistics this leads to a $\sqrt{10}$ improvement in energy resolution.