SPA 3609 Tutorial 2, Questions with outline answers for formative feedback

1. Estimate the ratio of the probability, per atom, of photoelectric absorption of a gamma ray in silicon to that in germanium.

    A) Probability varies ~ Z^5 so $(14/32)^5 = 0.016$

2. Indicate which of the three major processes (photoelectric, Compton, pair-production) is *dominant* in the following interactions of gamma rays:

    I. 1 MeV in aluminium

    II. 100 keV in hydrogen

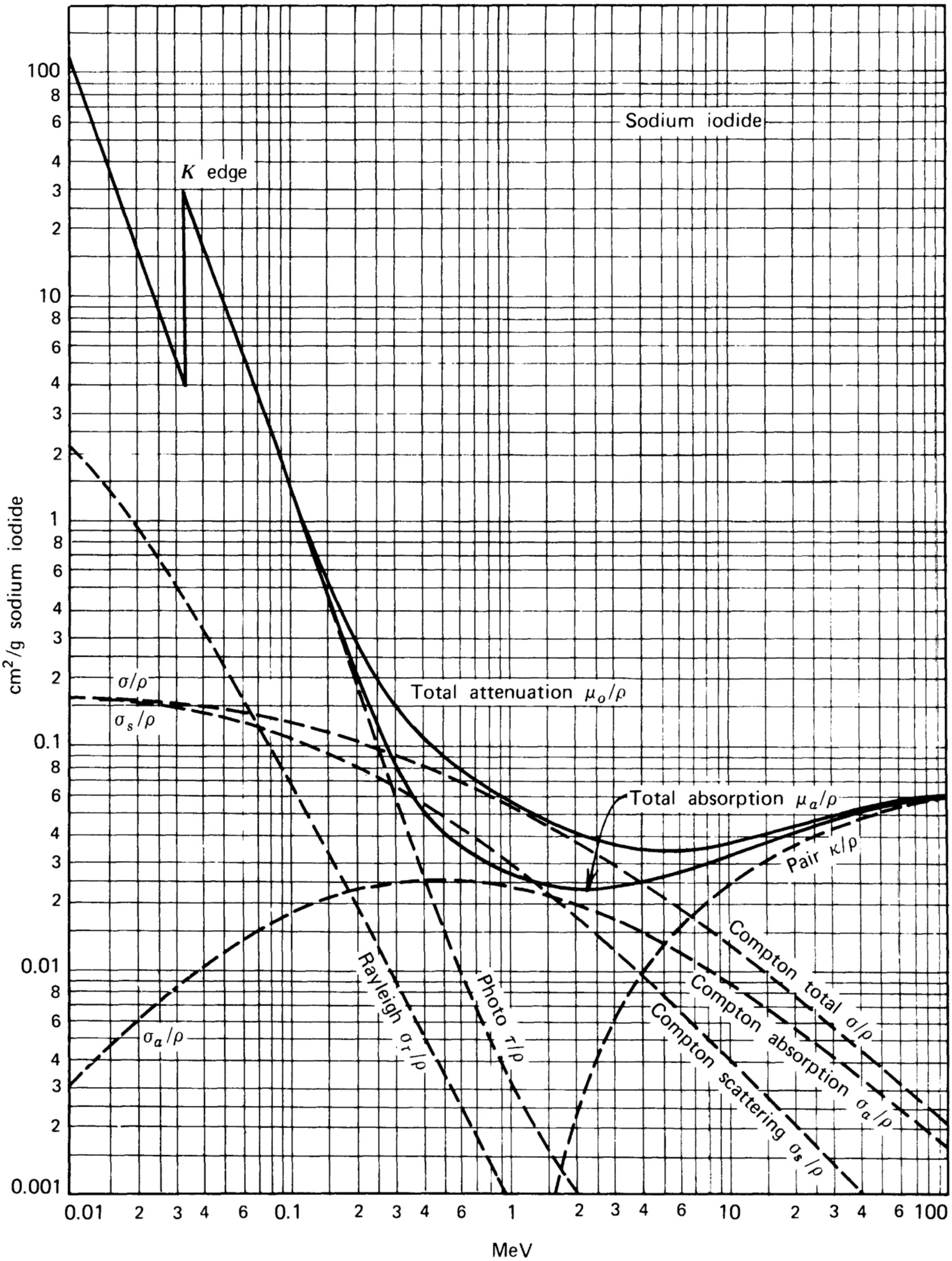
    III. 100 keV in iron

    IV. 10 MeV in carbon

    V. 10 MeV in lead

    A) I, II and IV: Compton, III: photoelectric, V: pair-production

3. Using the data in the figure (a) calculate the mean free path of 1 MeV gamma rays in NaI (ρ = 3.67 gcm^{-3}) and (b) determine the probability that a 600 keV gamma ray undergoes a *photoelectric* interaction in 1 cm of NaI.



Gamma ray interactions in NaI

(a). The gamma-ray mean free path (λ) in NaI is $1/\mu$ (where μ is the total linear attenuation coefficient in NaI). The mass attenuation coefficient ($\frac{\mu}{\rho}$) is 0.06 cm^2/gm at 1 MeV according to Figure 2.18, and the density of NaI relative to water (ρ) is 3.67 gm/cm^3(by the definition of specific gravity). Therefore, we have $\lambda = 1/\mu = \frac{1}{\left(\frac{\mu}{\rho}\right)*\rho}$. Here, we will denote the mass attenuation coefficient ($\frac{\mu}{\rho}$) by μ_ρ, so we have

$$\lambda = \frac{1}{(\mu_\rho \rho)}$$

We substitute $\mu_p = \frac{0.06\ \text{cm}^2}{g}$ and $\rho = \frac{3.67\ g}{\text{cm}^3}$to get the mean free path of 1 MeV gamma-rays in NaI (in cm).

λ **= 4.54 cm**

(b). Any photon which emerges from 1 cm cannot have undergone a photoelectric absorption. Neglecting buildup factors, the probability that a photon emerges from the slab without having an interaction is $e^{-\mu_T x}$, where $\mu_T$is the **total** attenuation coefficient. The complement of this is the probability that a photon doesn't emerge from the slab without having had at least one interaction (1-$e^{-\mu_T x}$). The probability that the interaction is a photoelectric interaction is τ/μ_T(this is not the probability per unit path length, but the total probability that any given interaction is a photoelectric interaction). Therefore, the probability that a photon undergoes photoelectric absorption in the slab is (τ/μ_T)*(1-$e^{-\mu_T x}$). This equation is expressed below, along with the values for μ_T (which is just 1 divided by the previous result for λ), the attenuation distance (denoted "**x**" and which is 1 cm), and τ, which is just the mass attenuation coefficient for photoelectric absorption (found on Figure 2.18 to be 0.01) multiplied by the density of NaI (3.67 g/cm3).

$$\textbf{Probability of photoelectric absorption} = \frac{\tau(1 - e^{-\mu_\tau x})}{\mu}$$

We substitute $\mu_\tau = \frac{1}{4.54\ \text{cm}}$, $x = 1$ cm and $\tau = \frac{0.01 \times 3.67}{\text{cm}}$ to get the probability of photoelectric absorption for 600 keV gamma-rays in 1 cm NaI.

Probability of photoelectric absorption = 0.0329

What is interesting is that a different result is obtained using a different, although seemingly equally valid approach. We can note that the probability per unit path length of a photoelectric interaction is τ, so $1 - e^{-\tau \mathbf{x}}$is the probability of a photoelectric interaction in traveling a distance **x**.

$$\textbf{Probability of } \boldsymbol{a} \textbf{ photoelectric interaction} = 1 - e^{-\tau x}$$

We substitute x=1cm and τ=$\frac{0.010 \times 3.67}{\text{cm}}$to get the probability of a photoelectric interaction in traveling a distance **x**.

Probability of *a* photoelectric interaction = 0.0360

This result is slightly (10%) larger from the previous answer because this approach does not account for the attenuation of photons through the material by other means.

4. A cylindrical proportional tube has a 60 μm diameter anode wire and a 4 cm diameter cathode. Assuming that it is operated at a potential difference of 2 kV and that a minimum electric field of 1 MV/m is needed for gas multiplication determine what fraction of the volume of the counter provides gas multiplication.

[You will need to remind yourself/look up the expression for the electric field from a cylindrical co-axial geometry, please note that in an examination I would *not* expect you to memorise or derive such an equation]

For a cylindrical wire, the electric field is given by $E(r) = \frac{V}{r \ln(b/a)}$. In this problem, we want to know what fraction of the tube has a field larger than a given value. Find the r for this electric field strength, then the fraction is just the ratio of the radii (r and b) squared. First, we solve for "r" in the above equation.

$$E = \frac{V}{r \ln\left(\frac{b}{a}\right)}$$

The equation for r:

$$r = \frac{V}{E \ln\left(\frac{b}{a}\right)}$$

Now we take the ratio of r^2 and b^2, substituting the known values V=2000Volts, a=0.003 cm, b=2cm and $V = \frac{10^6 \text{ Volts}}{\text{meter}}$ to get the percentage of the tube volume corresponding to the multiplication region.

$$\frac{r^2}{b^2} = 0.0237\ \%$$

This is a negligibly small fraction of the tube volume, which is a characteristic of the proportional tube.