SCHOOL OF PHYSICS & ASTRONOMY *QMUL*
SPA5302 *NUCLEAR PHYSICS & ASTROPHYSICS*

END OF TERM TEST 6 DECEMBER 2019

TIME AVAILABLE: 1 HOUR 30 MINUTES

Answer the questions in the spaces provided on these question sheets. If you run out of room for an answer, continue on the back of the page or at the end of the book.
Only non-programmable calculators may be used during this test.
This test has 13 pages of questions.
There are 60 marks available. FULL MARKS 50 = 100%

Name: ______________________

Student number: ______________________

Signature: ______________________

Do not write below this line

Question	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
8	5	
9	5	
10	5	
11	5	
12	5	
Total:	60	

USEFUL INFORMATION

The radius of a nuclei may be approximated by $R \approx 1.2A^{1/3}\,\mathrm{fm}$.
The semi-empirical mass formula (SEMF) for the binding energy of a nucleon is

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(Z,A)\,.$$

Constants in the SEMF: $a_V = 15.56, a_S = 17.23, a_C = 0.697, a_A = 23.28, a_P = 12.0$ where each number is in MeV.
Nuclear Shells: Protons

$$1s_{\frac{1}{2}} \Big\downarrow_{2} 1p_{\frac{3}{2}} \mid 1p_{\frac{1}{2}} \Big\downarrow_{8} 1d_{\frac{5}{2}} \mid 2s_{\frac{1}{2}} \mid 1d_{\frac{3}{2}} \Big\downarrow_{20} 1f_{\frac{7}{2}} \Big\downarrow_{28} 2p_{\frac{3}{2}} \mid 1f_{\frac{5}{2}} \mid 2p_{\frac{1}{2}} \mid 1g_{\frac{9}{2}} \Big\downarrow_{50} 1g_{\frac{7}{2}} \mid 2d_{\frac{5}{2}} \mid 1h_{\frac{11}{2}} \mid 2d_{\frac{3}{2}} \mid 3s_{\frac{1}{2}} \Big\downarrow_{82} 1h_{\frac{9}{2}} \mid 2f_{\frac{7}{2}}$$

Shells: Neutrons

$$1s_{\frac{1}{2}} \Big\downarrow_{2} 1p_{\frac{3}{2}} \mid 1p_{\frac{1}{2}} \Big\downarrow_{8} 1d_{\frac{5}{2}} \mid 2s_{\frac{1}{2}} \mid 1d_{\frac{3}{2}} \Big\downarrow_{20} 1f_{\frac{7}{2}} \Big\downarrow_{28} 2p_{\frac{3}{2}} \mid 1f_{\frac{5}{2}} \mid 2p_{\frac{1}{2}} \mid 1g_{\frac{9}{2}} \Big\downarrow_{50} 2d_{\frac{5}{2}} \mid 1g_{\frac{7}{2}} \mid 1h_{\frac{11}{2}} \mid 2d_{\frac{3}{2}} \mid 3s_{\frac{1}{2}} \Big\downarrow_{82} 2f_{\frac{7}{2}} \mid 1h_{\frac{9}{2}}$$

$\dfrac{e^2}{4\pi\varepsilon_0}$	$= 1.439965\,\mathrm{MeV\,fm}$
Boltzmann's constant	$k_B = 8.6173303 \times 10^{-5}\,\mathrm{eV/K}$
Planck's constant	$h = 4.135668 \times 10^{-15}\,\mathrm{eV\,s}$
Speed of light	$c = 2.99792 \times 10^{8}\,\mathrm{m/s}$
Neutron mean lifetime	$881\,\mathrm{s}$
Atomic mass unit	$1\,u = 931.4940954\,\mathrm{MeV}/c^2 = 1.66054 \times 10^{-27}\,\mathrm{kg}$
Mass of electron	$m_e = 5.4858 \times 10^{-4}\,u = 0.51099895\,\mathrm{MeV}/c^2$
Mass of proton	$m_p = 1.00727646688\,u = 938.27208\,\mathrm{MeV}/c^2$
Mass of neutron	$m_n = 1.00866491578\,u = 939.56541\,\mathrm{MeV}/c^2$
Mass of ${}^{1}_{1}\mathrm{H}$	$= 1.00782503\,u$
Mass of ${}^{2}_{1}\mathrm{H}$	$= 2.01410178\,u$
Mass of ${}^{3}_{1}\mathrm{H}$	$= 3.01604927\,u$
Mass of ${}^{3}_{2}\mathrm{He}$	$= 3.01602932\,u$
Mass of ${}^{4}_{2}\mathrm{He}$	$= 4.00260325\,u$
Mass of ${}^{232}_{90}\mathrm{Th}$	$= 232.038055\,u$
Mass of ${}^{234}_{90}\mathrm{Th}$	$= 234.043601\,u$
Mass of ${}^{235}_{92}\mathrm{U}$	$= 235.043930\,u$
Mass of ${}^{236}_{92}\mathrm{U}$	$= 236.045568\,u$
Mass of ${}^{238}_{92}\mathrm{U}$	$= 238.050788\,u$
Mass of ${}^{239}_{92}\mathrm{U}$	$= 239.054293\,u$
Mass of ${}^{240}_{94}\mathrm{Pu}$	$= 240.053811\,u$
Mass of ${}^{241}_{94}\mathrm{Pu}$	$= 241.056849\,u$
Mass of ${}^{242}_{94}\mathrm{Pu}$	$= 242.058741\,u$
Mass of the Sun	$M_\odot = 1.988 \times 10^{30}\,\mathrm{kg}$
Gravitational constant	$G = 6.67408 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$

Nuclei masses given are atomic masses.

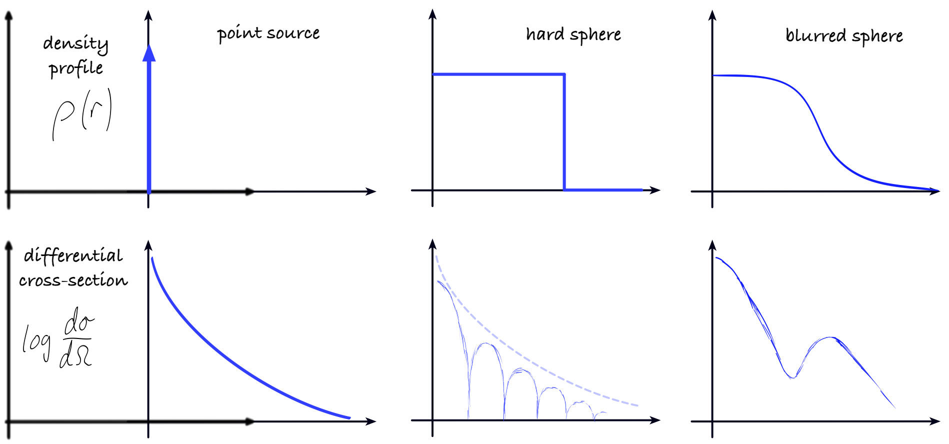
1. Charged particles are scattered off a heavy nucleus. Sketch a typical differential cross-section that would be observed, as a function of scattering angle. How does this compare to a cross-section that would be found by modelling the nucleus as 5

   a) a point particle, and

   b) a hard sphere?

   Sketch a realistic density profile for a nucleus.

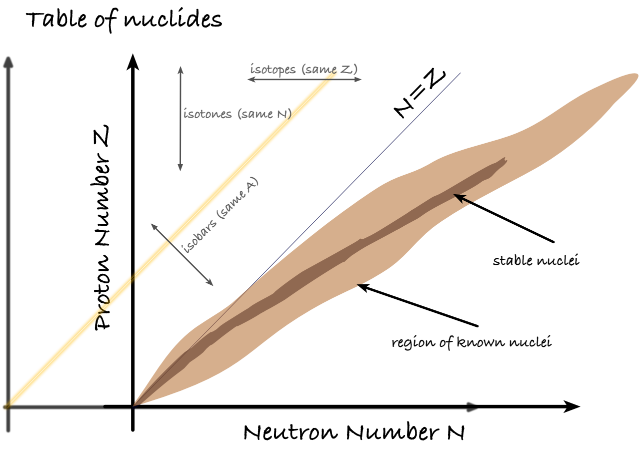
Solution:



Observed is closest to the last case.

2. Sketch a table of nuclides. Give the approximate positions of isotopes of Helium, Oxygen, and Thorium. In which regions are nuclei stable and unstable in this picture? 5

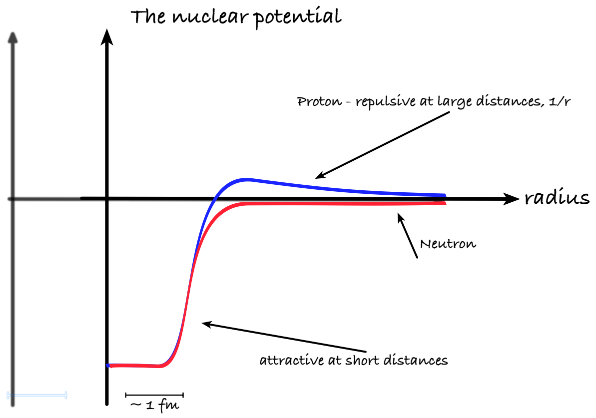
Solution:



EXTRA SPACE FOR ANSWERS

3. Sketch the nuclear potential for a neutron and a proton. Describe the key features of your plot. $\boxed{5}$

Solution:



4. Consider the semi-empirical mass formula (given on page 1). Discuss the form of the asymmetry term. $\boxed{5}$

Solution:

Asymmetry term: PEP: 2 identical fermions can't occupy same state.

each energy level can take only 2n + 2p (spin up and down). Lowest energy state when # of n and p are the same, which is more strongly bound. $(N-Z)^2$ accounts for this asymmetry - larger for more asymmetric numbers. (Arguing for the power is not necessary.) The $1/A$ factor comes from the approximate spacing between the energy levels $\sim$ 1/volume of potential well.

EXTRA SPACE FOR ANSWERS

5. Consider an isobar $A =$const. in the semi-empirical mass formula. Show there is a maximum binding energy when (ignore the pairing term and assume $Z(Z-1) \approx Z^2$) $\boxed{5}$

$$Z \approx \frac{A}{2}\frac{1}{1+\frac{1}{4}\frac{a_C}{a_A}A^{2/3}} \, .$$

Does this show that heavy nuclei have more protons or more neutrons?

Solution:

Let

$$B = \alpha + \beta Z - \gamma Z^2 \tag{1}$$

Where,

$$\alpha = a_v A - a_s A^{\frac{2}{3}} - a_A A, \quad \beta = 4a_A, \quad \gamma = \frac{a_c}{A^{\frac{1}{3}}} + \frac{4a_A}{A} \tag{2}$$

To find maximum binding energy, [1]

$$\frac{\partial B}{\partial Z} = 0 \tag{3}$$

Therefore

$$0 = \beta - 2\gamma Z \tag{4}$$

which implies, for Z_{min},

$$Z = \frac{\beta}{2\gamma} \approx \frac{4a_A}{\frac{8a_A}{A} + \frac{2a_c}{A^{\frac{1}{3}}}} \tag{5}$$

$$Z = \frac{4A}{8 + 2A^{\frac{2}{3}}\frac{a_c}{a_A}} \tag{6}$$

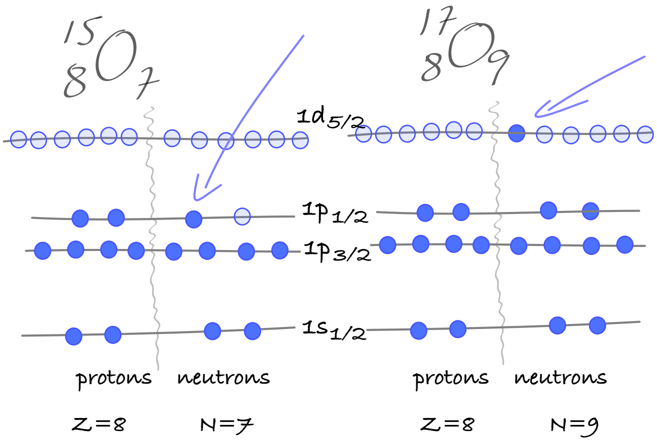
$$Z = \frac{A}{2}\frac{1}{1+\frac{1}{4}\frac{a_c}{a_A}A^{\frac{2}{3}}} \tag{7}$$

[3]

Since the denominator is > 1, heavy nuclei have more neutrons [1]

6. With the aid of a diagram, describe how to find the ground state of $^{29}_{14}\mathrm{Si}$. $\boxed{5}$

Solution: Figure something like



$^{29}_{14}$Si is $1/2^+$

EXTRA SPACE FOR ANSWERS

7. What are the nuclear magic numbers? Discuss 3 pieces of evidence for them. $\boxed{5}$

Solution: Numbers of N or Z = 2,8,20,28,50,82 where the binding energy is particularly strong (masses unusually light).

Usually the binding energy is very strong. The main features associated with these numbers are:

- Increased B if Z or N is magic.
- If N is magic there are more isotones.
- If Z is magic there are more isotopes.
- If both N and Z are magic, the nucleus is very stable.
- Elements with Z magic have higher natural abundances.
- Higher excitation energies.

8. Give the radioactive decay law for N nuclei with decay constant λ. An example of a decay chain is $\boxed{5}$

$$^{210}_{83}\mathrm{Bi} \longrightarrow {}^{210}_{84}\mathrm{Po} \longrightarrow {}^{206}_{82}\mathrm{Pb}\,.$$

What type of decay does each arrow correspond to?

Solution:

$N(t) = N_0 e^{-t/\tau}$ half-life is $t_{1/2} = \ln 2\,\tau$, $\lambda = \ln 2/\tau$.

β^- then α

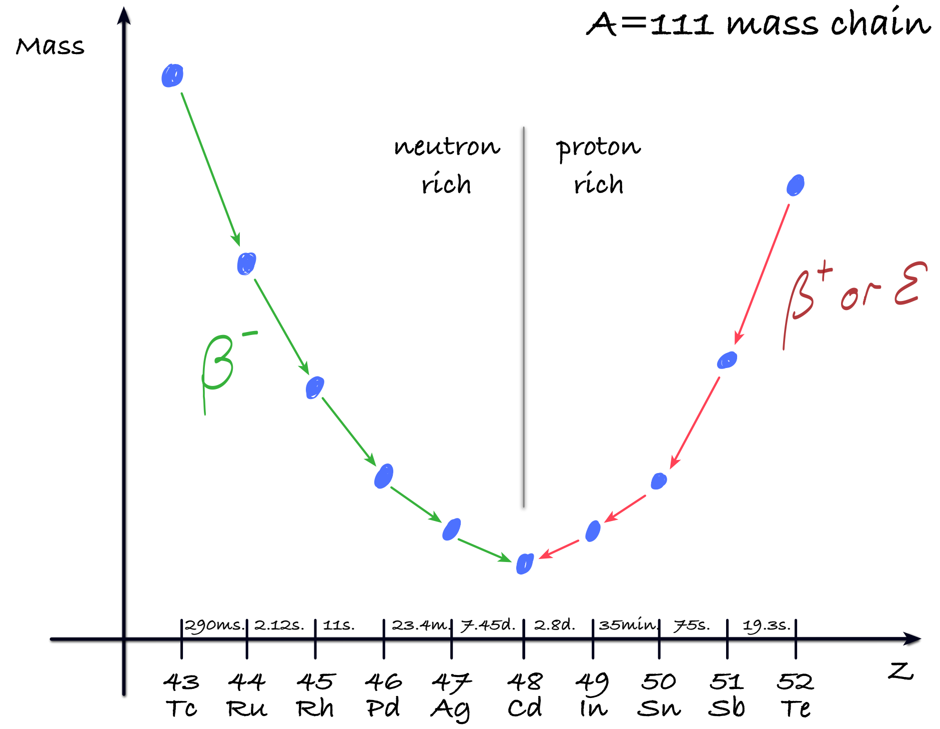
EXTRA SPACE FOR ANSWERS

9. For a nucleus (Z, A) give the reactions for the 3 types of possible β-decay. Discuss with the aid of a diagram how β-decay leads to the valley of stability in the table of nuclides. $\boxed{5}$

Solution: β^-: $n \to p + e^- + \bar{\nu}_e$. ${}^A_Z X \to^A_{Z+1} Y + e^- + \bar{\nu}_e$

β^+: $p \to n + e^+ + \nu_e$. ${}^A_Z X \to^A_{Z-1} Y + e^+ + \nu_e$

ε: $e^- + p \to n + \nu_e$. $e^- + {}^A_Z X \to^A_{Z-1} Y + \nu_e$



10. Given that the activation energy of ${}^{236}_{92}$U is 6.2 MeV, what is the minimum energy α-particle which can induce fission when hitting a ${}^{232}_{90}$Th target? Without knowing the mass, explain whether you expect this to be easier or harder to induce fission in ${}^{235}_{92}$U? $\boxed{5}$

Solution: $m(U) - m(Th) - m(He) = (236.045568 - 232.038055 - 4.00260325)931.5 = 4.57 MeV$ so the difference required is 1.63MeV.

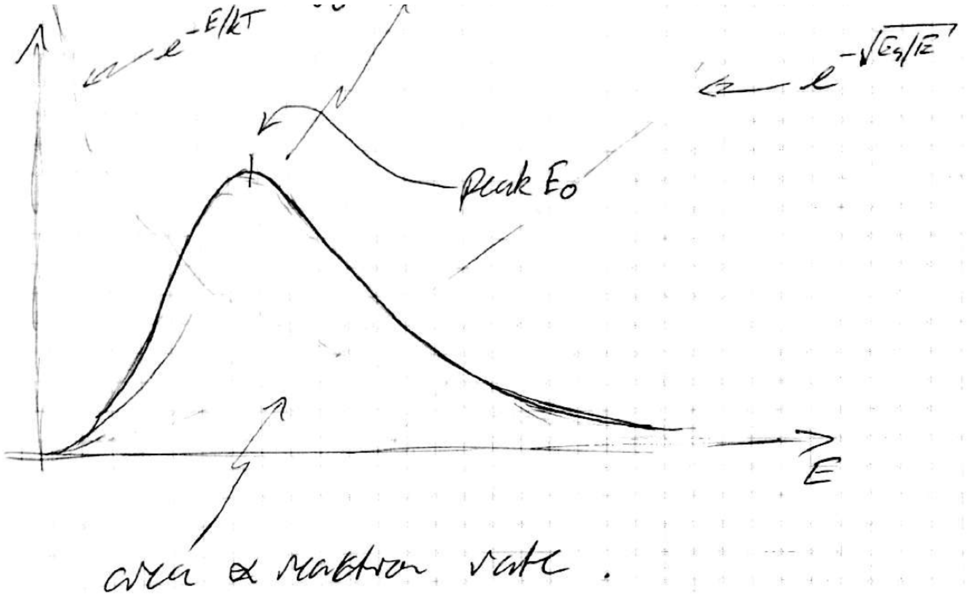
${}^{235}_{92}$U has an odd number of neutrons, so is less tightly bound than ${}^{236}_{92}$U (from the pairing term in the SEMF - nucleons like to pair), so inducing fission is easier.

EXTRA SPACE FOR ANSWERS

11. Consider a hot gas of nuclei at temperature T. One might expect that for fusion to be possible we need $T \sim V_C/k_B$, where V_C is the Coulomb barrier between the nuclei, and k_B is Boltzmann's constant. Give 2 reasons why fusion is possible in gases of much lower temperature. Illustrate your answer with a graph showing how the interplay of these two affects the reaction rate. $\boxed{5}$

Solution: Quantum tunnelling through the potential implies that the energy for fusion is much lower than a naive calculation of V_C suggests. The tunnelling probability behaves roughly as $\sim e^{-\sqrt{V_C/E}}$, which increases as the kinetic energy of the particles increases.

The velocity distribution of the particles is not distributed in a narrow range around kT, but has a long tail $\sim e^{-E/kT}$. These two competing effects mean that an optimal temperature for fusion occurs $\ll kT$, as shown in the graph



12. Give an account of the timeline involved in the production of Helium during Big Bang Nucleosynthesis. Helium production starts at $t \approx 300\,\mathrm{s}$ when the neutron fraction is 1/6. Hence show that the mass fraction of ^{4}He to Hydrogen is about 1/4. $\boxed{5}$

Solution: To create helium first need deuterium, which takes a long time to form. At 300s we have $n_n/n_p \sim 0.2e^{-300s/900s} \sim 0.15$. Since all the neutrons wind up in He, $n_{He}/n_p \sim n_n/n_p/2 \sim 0.07$. The mass fraction is 4 times this, which is about 0.25. (This is very approximate, more accuracy not required.)