



1. Derive the Gamow factor for quantum tunnelling for a fusion reaction given in lectures.
2. Show that matter radiation equality in the early universe happens at temperature

$$T_{eq} = \frac{T_0}{a_{eq}} \approx 6.5 \times 10^4 \Omega_m h^2 \text{ K},$$

3. Show that in the far future the expansion rate will be constant and that the scale factor will grow exponentially.
4. Argue that at very early times the cosmological scale factor grows as the square root of the time. Hence show

$$T(t) \sim 10^{10} \text{ K} \left(\frac{1 \text{ s}}{t} \right)^{1/2}.$$

5. For $t \ll 1 \text{ s}$ write down the weak interactions which keep the protons and neutrons in equilibrium.
6. If neutron freeze-out happens at $kT \sim 0.8 \text{ MeV}$, what is the neutron fraction? If the mass of the neutron were much closer to the mass of the proton, would this fraction be more or less?
7. Show that for the formation of deuterium, the temperature T given by the solution to

$$\left(\frac{kT}{m_p c^2} \right)^{3/2} e^{B_D/kT} \sim \frac{1}{\eta},$$

must be reached. The solution to this is $kT \sim 0.06 \text{ MeV}$ for $\eta \sim 6 \times 10^{-10}$. If the baryon to photon ratio was much larger, would you expect this to be much higher or lower? Would there be more helium or less?

Some (potentially) useful information:

The radius of a nuclei may be approximated by $R \approx 1.2A^{1/3}$ fm.
 The semi-empirical mass formula (SEMF) for the binding energy of a nucleon is

$$B(Z, A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(Z, A).$$

Constants in the SEMF: $a_V = 15.56, a_S = 17.23, a_C = 0.697, a_A = 23.28, a_P = 12.0$ where each number is in MeV.

Nuclear Shells: Protons

$$1s_{\frac{1}{2}} \downarrow_2 \quad 1p_{\frac{3}{2}} \downarrow_4 \quad 1p_{\frac{1}{2}} \downarrow_2 \quad 1d_{\frac{5}{2}} \downarrow_6 \quad 2s_{\frac{1}{2}} \downarrow_2 \quad 1d_{\frac{3}{2}} \downarrow_4 \quad 1f_{\frac{7}{2}} \downarrow_8 \quad 2p_{\frac{3}{2}} \downarrow_4 \quad 1f_{\frac{5}{2}} \downarrow_6 \quad 2p_{\frac{1}{2}} \downarrow_2 \quad 1g_{\frac{9}{2}} \downarrow_{10} \quad 1g_{\frac{7}{2}} \downarrow_8 \quad 2d_{\frac{5}{2}} \downarrow_6 \quad 1h_{\frac{11}{2}} \downarrow_{10} \quad 2d_{\frac{3}{2}} \downarrow_4 \quad 3s_{\frac{1}{2}} \downarrow_2 \quad 1h_{\frac{9}{2}} \downarrow_8 \quad 2f_{\frac{7}{2}} \downarrow_6 \quad \dots$$

Shells: Neutrons

$$1s_{\frac{1}{2}} \downarrow_2 \quad 1p_{\frac{3}{2}} \downarrow_4 \quad 1p_{\frac{1}{2}} \downarrow_2 \quad 1d_{\frac{5}{2}} \downarrow_6 \quad 2s_{\frac{1}{2}} \downarrow_2 \quad 1d_{\frac{3}{2}} \downarrow_4 \quad 1f_{\frac{7}{2}} \downarrow_8 \quad 2p_{\frac{3}{2}} \downarrow_4 \quad 1f_{\frac{5}{2}} \downarrow_6 \quad 2p_{\frac{1}{2}} \downarrow_2 \quad 1g_{\frac{9}{2}} \downarrow_{10} \quad 2d_{\frac{5}{2}} \downarrow_6 \quad 1g_{\frac{7}{2}} \downarrow_8 \quad 1h_{\frac{11}{2}} \downarrow_{10} \quad 2d_{\frac{3}{2}} \downarrow_4 \quad 3s_{\frac{1}{2}} \downarrow_2 \quad 2f_{\frac{7}{2}} \downarrow_6 \quad 1h_{\frac{9}{2}} \downarrow_8 \quad \dots$$

$\frac{e^2}{4\pi\epsilon_0}$	= 1.439965 MeV fm
Boltzmann's constant	$k_B = 8.6173303 \times 10^{-5}$ eV/K
Planck's constant	$h = 4.135668 \times 10^{-15}$ eV s
Speed of light	$c = 2.99792 \times 10^8$ m/s
Neutrino mean lifetime	881 s
Atomic mass unit	$1 u = 931.4940954 \text{ MeV}/c^2 = 1.66054 \times 10^{-27}$ kg
Mass of electron	$m_e = 5.4858 \times 10^{-4} u = 0.51099895 \text{ MeV}/c^2$
Mass of proton	$m_p = 1.00727646688 u = 938.27208 \text{ MeV}/c^2$
Mass of neutron	$m_n = 1.00866491578 u = 939.56541 \text{ MeV}/c^2$
Mass of ^1_1H	= 1.00782503 u
Mass of ^2_1H	= 2.01410178 u
Mass of ^3_1H	= 3.01604927 u
Mass of ^3_2He	= 3.01602932 u
Mass of ^4_2He	= 4.00260325 u
Mass of $^{232}_{90}\text{Th}$	= 232.038055 u
Mass of $^{234}_{90}\text{Th}$	= 234.043601 u
Mass of $^{235}_{92}\text{U}$	= 235.043930 u
Mass of $^{236}_{92}\text{U}$	= 236.045568 u
Mass of $^{238}_{92}\text{U}$	= 238.050788 u
Mass of $^{239}_{92}\text{U}$	= 239.054293 u
Mass of $^{240}_{94}\text{Pu}$	= 240.053811 u
Mass of $^{241}_{94}\text{Pu}$	= 241.056849 u
Mass of $^{242}_{94}\text{Pu}$	= 242.058741 u
Mass of the Sun	$M_\odot = 1.988 \times 10^{30}$ kg
Gravitational constant	$G = 6.67408 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$

Nuclei masses given are atomic masses.

You can look up other nuclear data from websites

- <https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html>
- <http://www.nndc.bnl.gov/nudat2/>
- <http://atom.kaeri.re.kr/nuchart/>
- <http://people.physics.anu.edu.au/~ecs103/chart/>