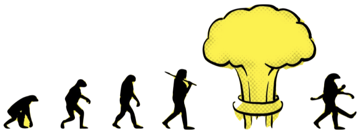

SPA5302
EXERCISE SHEET 5

NUCLEAR PHYSICS & ASTROPHYSICS
2019

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1. Consider the Q values for β-decay. For a nucleus (Z, A) derive formulas for the Q values in terms of *atomic* masses and *nuclear* masses for each type of β-decay.

Solution: use $m(Z, A) = M_N(Z, A) + Zm_e$, ignoring the binding energy of the electrons. Also ignore the mass of the neutrino, and that the positron mass is that of the electron. Then

$$\begin{aligned} Q_{\beta^-} &= [m(Z,A) - m(Z+1,A)]c^2 \\ Q_{\beta^+} &= [m(Z,A) - m(Z-1,A) - 2m_e]c^2 \\ Q_{\varepsilon} &= [m(Z,A) - m(Z-1,A)]c^2 \end{aligned}$$

2. A free neutron undergoes β^- decay. Write down this decay, and calculate the Q-value. What will the kinetic energy of the resulting proton be in the case where the the momentum of the neutrino is negligible? [Hint: the electron is relativistic!]

Solution:

From conservation of momentum we know that:

$$p_p + p_e = 0 \tag{1}$$

Therefore we can say that $p_e = -p_p$. Then using the conervation of energy, and the fact that the electron is relativistic we get:

$$T_p + T_e = Q \tag{2}$$

$$\frac{1}{2}mv_p^2 + p_e c = Q \tag{3}$$

$$\frac{1}{2}mv_p^2 - p_p c = Q \tag{4}$$

$$\frac{1}{2}mv_p^2 - mv_p c = Q. \tag{5}$$

We know that $\frac{1}{2}mv_p^2 = T_p$, therefore $v_p = \sqrt{\frac{2T_p}{m}}$. Substituting this in above we get:

$$T_p - mc\sqrt{\frac{2T_p}{m}} = Q \tag{6}$$

$$mc\sqrt{\frac{2T_p}{m}} = T_p - Q \tag{7}$$

$$m^2c^2\frac{2T_p}{m} = (T_p - Q)^2 \tag{8}$$

$$mc^2 2T_p = T_p^2 + Q^2 - 2T_pQ \tag{9}$$

$$T_p^2 - (2mc^2 - 2Q)T_p + Q^2 = 0 \tag{10}$$

$$T_p = \frac{(2mc^2 + 2Q) \pm \sqrt{(2mc^2 + 2Q)^2 - 4Q^2}}{2} \tag{11}$$

$$T_p = (mc^2 + Q) \pm \sqrt{m^2c^4 + 2mc^2Q} \tag{12}$$

Substituting in $m = m_p$ and $Q \approx 0.782 MeV$, we get:

$$T_p \approx 0.32 KeV. \tag{13}$$

3. In the lectures we sketched a diagram (called a Feynman diagram) for β^- decay, where a d quark was converted to a u quark. Try sketch one for β^+ (you can use google!).

Solution: Same as in the notes - but switch the signs on the W and e, swap d and u on the interacting quark, remove the bar on $\bar{\nu}$ and reverse the arrows on the e and ν

4. The cross section for a neutrino from β-decay is $\sim 10^{-47}\text{m}^2$. With this nominal cross section, some estimates of rates of interaction can be made. Multiplying the cross section times the density of a substance in atomic mass units gives a number of interactions per meter, and the inverse of that is an estimate of the mean free path. What is the mean free path of a neutrino in water of density $1000\,\text{kg/m}^3$? (Write your answer in light-years.) In a bathtub of water, what percentage of neutrinos would interact at all with the water?

Solution: The mean free path is found from $1u/\sigma/$(density of water in kg/m^3) 10^{17}m which is about 100 light years. So, only on 1 in 10^{17} neutrinos would interact with a small tank of water.

See `http://hyperphysics.phy-astr.gsu.edu/hbase/Particles/neutrino3.html` for more discussion.

5. Identify the important terms in the SEMF which govern spontaneous fission. Discuss the competing physical effects taking place. Why would a ball of neutrons be very unstable to fission?

Solution:

It's a competition between the surface term, which acts to push the nucleus into a spherical shape [from the nuclear force], and the Coulomb term, which acts to break a nucleus apart. Because the nuclear force goes to zero at large distances, for very heavy nuclei the Coulomb repulsion eventually overcomes the nuclear force, fracturing the nucleus. Review the calculation of this in the notes.

The argument is slightly different for very neutron rich nuclei as the asymmetry term becomes very important, and makes the nuclei unstable. In fact you can see that as $Z \to 0$ the binding energy becomes negative.

6. Given that the activation energy of ^{236}U is 6.2 MeV what is the minimum energy α-particle which can induce fission when hitting a ^{232}Th target?

Solution: $m(U) - m(Th) - m(He) = (236.045568 - 232.038055 - 4.00260325)931.5 = 4.57MeV$ so the difference required is 1.63MeV.

7. (Non-examinable material) On QM+ there is a resource paper on the derivation of the 'Bohr-Wheeler' spontaneous fission limit – we quoted simplified results in the lectures. Work through this calculation.

8. In the lectures we have shown that barrier penetration is more difficult for fission than it is for α-decay. Given that the Gamow factor is proportional to the integral $I = \int dr \sqrt{V(r) - Q}$, write down the tunnelling probability in terms of I. Consider a heavy nucleus with $A = 200$ that undergoes decay into two lighter nuclei with $A = 100$. How does this probability compare to the tunnelling probability of α-decay, assuming the value of I is the same in both cases? (You only need to provide a rough answer).

Solution:

Probability is $P = e^{-2G}$ with $G = \sqrt{2m/\hbar^2} I$. Roughly $m(Z,4)/m(Z,100) \sim 4/100 \sim 1/25$ (very roughly!), so the differences in G is $1/\sqrt{25} \sim 1/5$ so that $P_{\text{fission}} \sim P_\alpha^5$.

Some (potentially) useful information:

The radius of a nuclei may be approximated by $R \approx 1.2A^{1/3}$ fm.
The semi-empirical mass formula (SEMF) for the binding energy of a nucleon is

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(Z,A)\,.$$

Constants in the SEMF: $a_V = 15.56, a_S = 17.23, a_C = 0.697, a_A = 23.28, a_P = 12.0$ where each number is in MeV.

Nuclear Shells: Protons

$$1s_{\frac{1}{2}} \Big\downarrow_{2} 1p_{\frac{3}{2}} \mid 1p_{\frac{1}{2}} \Big\downarrow_{8} 1d_{\frac{5}{2}} \mid 2s_{\frac{1}{2}} \mid 1d_{\frac{3}{2}} \Big\downarrow_{20} 1f_{\frac{7}{2}} \Big\downarrow_{28} 2p_{\frac{3}{2}} \mid 1f_{\frac{5}{2}} \mid 2p_{\frac{1}{2}} \mid 1g_{\frac{9}{2}} \Big\downarrow_{50} 1g_{\frac{7}{2}} \mid 2d_{\frac{5}{2}} \mid 1h_{\frac{11}{2}} \mid 2d_{\frac{3}{2}} \mid 3s_{\frac{1}{2}} \Big\downarrow_{82} 1h_{\frac{9}{2}} \mid 2f_{\frac{7}{2}} \mid \cdots$$

Shells: Neutrons

$$1s_{\frac{1}{2}} \Big\downarrow_{2} 1p_{\frac{3}{2}} \mid 1p_{\frac{1}{2}} \Big\downarrow_{8} 1d_{\frac{5}{2}} \mid 2s_{\frac{1}{2}} \mid 1d_{\frac{3}{2}} \Big\downarrow_{20} 1f_{\frac{7}{2}} \Big\downarrow_{28} 2p_{\frac{3}{2}} \mid 1f_{\frac{5}{2}} \mid 2p_{\frac{1}{2}} \mid 1g_{\frac{9}{2}} \Big\downarrow_{50} 2d_{\frac{5}{2}} \mid 1g_{\frac{7}{2}} \mid 1h_{\frac{11}{2}} \mid 2d_{\frac{3}{2}} \mid 3s_{\frac{1}{2}} \Big\downarrow_{82} 2f_{\frac{7}{2}} \mid 1h_{\frac{9}{2}} \mid \cdots$$

$\frac{e^2}{4\pi\varepsilon_0}$	$= 1.439965\,\text{MeV fm}$
Boltzmann's constant	$k_B = 8.6173303 \times 10^{-5}\,\text{eV/K}$
Planck's constant	$h = 4.135668 \times 10^{-15}\,\text{eV s}$
Speed of light	$c = 2.99792 \times 10^{8}\,\text{m/s}$
Neutron mean lifetime	881 s
Atomic mass unit	$1\,u = 931.4940954\,\text{MeV}/c^2 = 1.66054 \times 10^{-27}\,\text{kg}$
Mass of electron	$m_e = 5.4858 \times 10^{-4}\,u = 0.51099895\,\text{MeV}/c^2$
Mass of proton	$m_p = 1.00727646688\,u = 938.27208\,\text{MeV}/c^2$
Mass of neutron	$m_n = 1.00866491578\,u = 939.56541\,\text{MeV}/c^2$
Mass of $^{1}_{1}\text{H}$	$= 1.00782503\,u$
Mass of $^{2}_{1}\text{H}$	$= 2.01410178\,u$
Mass of $^{3}_{1}\text{H}$	$= 3.01604927\,u$
Mass of $^{3}_{2}\text{He}$	$= 3.01602932\,u$
Mass of $^{4}_{2}\text{He}$	$= 4.00260325\,u$
Mass of $^{232}_{90}\text{Th}$	$= 232.038055\,u$
Mass of $^{234}_{90}\text{Th}$	$= 234.043601\,u$
Mass of $^{235}_{92}\text{U}$	$= 235.043930\,u$
Mass of $^{236}_{92}\text{U}$	$= 236.045568\,u$
Mass of $^{238}_{92}\text{U}$	$= 238.050788\,u$
Mass of $^{239}_{92}\text{U}$	$= 239.054293\,u$
Mass of $^{240}_{94}\text{Pu}$	$= 240.053811\,u$
Mass of $^{241}_{94}\text{Pu}$	$= 241.056849\,u$
Mass of $^{242}_{94}\text{Pu}$	$= 242.058741\,u$
Mass of the Sun	$M_\odot = 1.988 \times 10^{30}\,\text{kg}$
Gravitational constant	$G = 6.67408 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$

Nuclei masses given are atomic masses.

You can look up other nuclear data from websites
`https://www-nds.iaea.org/relnsd/vcharthtml/VChartHTML.html`
`http://www.nndc.bnl.gov/nudat2/`
`http://atom.kaeri.re.kr/nuchart/`
`http://people.physics.anu.edu.au/~ecs103/chart/`