Physical Cosmology Revision Checklist (January 2020 final exam)

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General exam tips

The best way to practise is to do the problem sheets and revision sheets, and to re-do the mid-term exam(s). The assessed questions on the problem sheets are generally a bit more difficult than exam questions, so don't worry about these too much. The non-assessed questions have more in common with typical Part B exam questions (although the format is different, so don't take this too seriously). The revision sheet questions are more similar to Part A questions. The mid-term exam will be good practice for Part A and some bits and pieces of Part B.

Complicated equations will generally be given to you in the question – for example, in addition to the formulae on that sheet, you won't need to memorise the Growth equation, Klein-Gordon equation, Raychaudhuri equation, Saha equation, blackbody (Planck) distribution, deceleration parameter definition, or FLRW line element. Make sure that you understand those equations and how to use them though; for example, you should be able to do things like:

- Convert between the two forms of the Friedmann equation
- Solve the Friedmann equation for many different types of matter/energy
- Use the conservation equation to derive the equation of state
- Simplify the Raychaudhuri equation depending on what types of matter/energy are present
- Derive the comoving distance $r(z)$ from the FLRW line element
- Calculate the Hubble radius
- Calculate the expansion rate H by using the Friedmann equation
- Calculate the redshift of matter-radiation equality and age of the Universe
- Convert between coordinates, e.g. between a, t, and z, or to find quantities like dr/dz
- Apply the slow-roll approximation to the Klein-Gordon equation
- Evaluate the derivatives in the growth equation

There is no need to memorise any of the more complicated derivations from the notes, like the Newtonian derivation of the Friedmann equation, and you won't need to use the tensor form of the FLRW metric either.

You are expected to remember simpler definitions, like the critical density, Omega parameters (especially Ω_k), the equation of state of different types of matter/energy, the angular diameter distance and luminosity distance, how temperature scales with redshift, the density contrast, and of course how redshift and the scale factor are related to each other (and wavelength/frequency of light).

You should be able to sketch (and label!) several of the important quantities that we've come across, such as the blackbody spectrum, the Hubble radius as a function of time/scale factor, the CMB power spectrum, galaxy correlation function, and simple inflationary potentials. Remember to label your axes too...

Also, make sure you can give sensible qualitative explanations of the various physical phenomena we've come across, such as:

- How cosmic expansion and peculiar velocities affect the wavelength of light
- The particle horizon and Hubble radius
- Recombination, and why it happens at much lower temperatures than the ionisation energy of hydrogen
- Decoupling, and how it depends on the mean-free path of photons
- What happens at last-scattering (incl. how the mean-free path of photons changes)

- What causes temperature anisotropies in the CMB
- Baryon acoustic oscillations
- Where the features in the CMB power spectrum came from (e.g. Sachs-Wolfe, diffusion damping)
- Problems solved by inflation (horizon, flatness, monopole problems)
- How inflation works, and how it solves the three problems above
- What the slow-roll approximation is, and why it is used
- Observational evidence for dark matter
- How the temperature of dark matter affects the large-scale structure (e.g. halo mass function, abundance of dwarf galaxies)
- What the galaxy correlation function is

Finally, for clarity: the sections in the notes about Big Bang Nucleosynthesis and Observational Cosmology will not be examinable.

1. Expanding universe

- What is cosmology?
- What are the major epochs in the Universe's history?
- What physical processes are important during each of these epochs?
- What does it mean for space to be expanding?
- What does it mean for a universe to be homogeneous and isotropic?
- What is Olbers' paradox, and how is it resolved?
- How does the expansion of space affect the frequency of EM radiation?
- How are spectra used to measure redshift?
- What is the evidence for the hot Big Bang model?

2. Geometry and distance

- What are the definitions of 'recession velocity' and 'Hubble's Law'?
- What are proper and comoving coordinates?
- What is the FLRW metric and the corresponding line element for an expanding universe?
- How can a Taylor series be used to approximate a function?
- What does it mean for space to be curved?
- What are the different types of curved spaces, and what are their properties?

3. Friedmann equation

- What is the Friedmann equation?
- What does each term in the Friedmann equation mean, and what are the relevant units?
- What is the critical density and how can it be used to rewrite the Friedmann equation?
- How do the densities of matter and radiation depend on the scale factor?
- How can you solve the Friedmann equation to find the scale factor as a function of time?
- How can you solve the Friedmann equation to find the age of the Universe?
- How does curvature affect the time evolution of the scale factor?

4. Distances and horizons

- What is the comoving distance, $r(z)$?
- How is the luminosity distance defined?
- How is the angular diameter distance defined?
- How are the luminosity distance and angular diameter distance related?
- How can the different types of distance be measured?
- What is the particle horizon? How can it be calculated?
- What is the Hubble radius or Hubble horizon?

5. Cosmic acceleration

- What is the conservation equation?
- How can it be used to work out how the density of a type of matter/energy depends on scale factor?
- What is the Raychaudhuri equation?
- How does the equation of state relate energy density and relativistic pressure?
- What is the equation of state for matter, radiation, and a cosmological constant?
- What does it mean to have accelerating expansion?
- How is the deceleration parameter defined?
- What is the Cosmological Constant?
- What is the Cosmological Constant problem?
- How does the Hubble horizon behave during exponential expansion?

6. CMB radiation

- What is the Cosmic Microwave Background (CMB)?
- What was the Universe like before the CMB formed?
- What is the mean free path of a photon in an ionised medium?
- What are decoupling and recombination?
- What roles did decoupling and recombination play in the formation of the CMB?
- What is the last scattering surface of the CMB? How is it defined for different observers?
- What is a blackbody frequency spectrum?
- How does the temperature of a blackbody spectrum vary with redshift?
- At approximately what redshift did the CMB form?

7. CMB anisotropies

- What are CMB anisotropies?
- What physical effects cause CMB anisotropies?
- What are the baryon acoustic oscillations, how do they form, and what do they look like in the CMB power spectrum?
- What is the approximate comoving size of the BAO feature?
- What is diffusion damping?
- What are the Sachs-Wolfe and integrated Sachs-Wolfe effects?
- What are spherical harmonics, and why are they used to analyse the CMB?
- What patterns do spherical harmonics make on the sky?
- What is the relationship between spherical harmonic mode, ℓ, and angle on the sky, θ?
- What is a power spectrum? Why do we use it to study the CMB anisotropies?
- What does the CMB power spectrum look like, and where do its various features come from?
- What is the acoustic peak and how can it be used to measure the distance to the CMB?

8. Inflation

- What are the horizon, flatness, and monopole problems?
- What is the inflationary mechanism?
- How does inflation solve these problems?
- What is the inflaton and what are its properties?
- What is an e-fold?
- What does the cosmological Klein-Gordon equation describe?
- How are the density and pressure of the inflaton related to its kinetic energy and potential?
- What is the slow-roll approximation?
- How does inflation generate primordial fluctuations?

9. Dark matter

- What is dark matter?
- What are the three main pieces of observational evidence for dark matter?
- What are the known properties of dark matter?
- What are some of the proposals for what dark matter might be made of?
- What are WIMPs and how can they be detected?
- What are MACHOs and how can they be detected?
- What are the differences between hot, warm, and cold dark matter?
- What is the difference between a bound and unbound structure?
- What is the difference between top-down and bottom-up structure formation?
- What is a dark matter halo?
- What is one piece of evidence that disfavours WDM compared to CDM?

10. Structure formation

- What does it mean to perturb the metric?
- What is the definition of the density contrast?
- What is the growth factor, D, and growth rate, f?
- How does dark energy affect the growth rate?
- What is the matter power spectrum and how is it related to the density contrast?
- What is galaxy bias?
- What is the galaxy correlation function and how is it related to the matter correlation function?
- What does the Baryon Acoustic Oscillation feature look like in the galaxy correlation function?
- How can Fourier transforms be used to simplify perturbation equations?
- How is the velocity related to the density contrast in Fourier space?