Revision Questions II

Week 6: CMB radiation

1. Write down how temperature depends on redshift.
2. Define the terms *decoupling*, *recombination*, and *last scattering*.
3. How does the mean-free path of photons change at last scattering?
4. Explain the process of decoupling.
5. Would decoupling have happened earlier or later in the Universe's history if the number density of electrons, n_e, had been larger?
6. Calculate the approximate energy, in eV, of a photon with temperature $T = 4000$ K.
7. Explain the process of recombination.
8. Explain why recombination happens at a lower energy than 13.6 eV (the ionisation energy of hydrogen).
9. Define the *last-scattering surface*.
10. What would be the redshift of the last-scattering surface if the CMB had been emitted at a temperature of $T = 40,000$ K? (The observed CMB temperature today is $T_0 = 2.725$ K.)
11. How would a Doppler shift due to the peculiar velocity of our galaxy affect the observed CMB temperature?

Week 8: CMB anisotropies

1. Find the approximate angular size, in degrees, of CMB anisotropies observed at $\ell = 300$.
2. Briefly explain what is meant by a *temperature anisotropy* in the CMB radiation?
3. Explain how regions of the last scattering surface with higher than average density affect the observed CMB temperature.
4. Explain how the velocity of matter at the last scattering surface affects the CMB temperature.
5. Explain the difference between the Sachs-Wolfe and integrated Sachs-Wolfe effects. Which is a primary anisotropy, and which is a secondary anisotropy?
6. Sketch the CMB power spectrum as a function of spherical harmonic wavenumber, ℓ. Mark the approximate position of the acoustic peak.
7. On your sketch of the CMB power spectrum, label the features caused by the Sachs-Wolfe effect, integrated Sachs-Wolfe effect, and baryon acoustic oscillations.
8. Explain why the CMB power spectrum drops as ℓ increases.
9. Explain how the baryon acoustic oscillations formed.
10. Explain how the integrated Sachs-Wolfe effect provides evidence for cosmic acceleration.
11. Explain how the baryon acoustic oscillations provide evidence for the existence of dark matter.

Week 9: Inflation

1. Explain the horizon problem. Refer to at least one piece of observational evidence in your answer.
2. Explain the flatness problem. Refer to at least one piece of observational evidence in your answer.
3. Explain the monopole problem. Refer to at least one piece of observational evidence in your answer.
4. Show that a universe with $w = -1$ leads to exponential expansion, $a(t) \propto e^{Ht}$.
5. Show that $H \approx \text{const.}$ in an exponentially-expanding universe.
6. Show that the Hubble radius decreases with time in an exponentially-expanding universe.
7. Sketch the Hubble radius in (i) an exponentially-expanding universe, and (ii) a radiation-only universe, as a function of scale factor.
8. Explain how inflation solves the horizon and monopole problems.
9. Explain how inflation solves the flatness problem.
10. Write down expressions for the energy density and relativistic pressure of a scalar field, ϕ.
11. Show that, in the slow-roll approximation, the equation of state of the scalar field is $w_\phi \approx -1$.
12. Write down the Klein-Gordon equation in the slow-roll approximation. Justify why you have kept or neglected each term.
13. Write down the Friedmann equation in the slow-roll approximation, in a flat universe containing only a scalar field.
14. Sketch a potential, $V(\phi)$, that would allow slow-roll inflation to occur.

Week 11: Dark matter

1. Name three pieces of observational evidence for dark matter.
2. Explain why each piece of evidence from the question above implies the existence of hidden (invisible) mass.
3. State three known properties of dark matter.
4. Explain the difference between particle dark matter and baryonic dark matter.
5. Name one piece of evidence that disfavours baryonic dark matter.
6. Explain the difference between hot dark matter, warm dark matter, and cold dark matter.
7. Explain how the temperature of dark matter affects the formation of structures.
8. Describe one piece of evidence that favours CDM over WDM.

Week 12: Structure formation

1. Briefly define each kind of large-scale structure: halos/clusters, voids, filaments.
2. Write down the (mathematical) definition of the density contrast, δ.
3. Define the growth factor, $D(a)$, in terms of the density contrast at different times.
4. Define the growth rate, f, in terms of derivatives of the growth factor.
5. Show that $D \propto a$ is a solution to the growth equation if the universe contains only matter.
6. Calculate $f(a)$ if $D \propto a$.

7. Briefly explain how the expansion of space affects the growth of structure.
8. Define the matter power spectrum, $P(k)$, in terms of the variance of the density contrast.
9. Evaluate $\int \delta^{(3)}(\vec{x} - \vec{x}')f(\vec{x})g(\vec{x}')d^3x$.
10. Sketch the correlation function, $\xi(r)$. Label the baryon acoustic oscillation feature, including its approximate location in Mpc.
11. Explain how the galaxy correlation function, $\xi_g(r)$, depends on the galaxy bias, b.
12. Explain what the correlation function tells us about the distribution of galaxies.