Week 12: Dark matter and structure formation

This sheet doesn't need to be handed in for marking.

1. **Dark matter**
   (a) Name three pieces of observational evidence for dark matter.
   (b) Explain the difference between particle dark matter and baryonic dark matter.
   (c) Define *hot dark matter*, *warm dark matter*, and *cold dark matter*.
   (d) Explain how the halo mass function differs between the WDM and CDM scenarios. Make reference to the mean free path of dark matter particles in your answer.

2. **Galaxy correlation function**
   (a) Briefly explain what the galaxy correlation function, $\xi_g(r)$, tells us about pairs of galaxies.
   (b) Write down the relationship between the matter correlation function and the galaxy correlation function. Define the parameter b in this expression.
   (c) Sketch the galaxy correlation function. Label your sketch with the approximate position of the BAO feature, in Mpc.

3. **Definition of the power spectrum**
   The matter power spectrum, $P(k)$, is defined through the expression $\langle \delta(\vec{k})\delta^*(\vec{k}')\rangle = (2\pi)^3\delta^{(3)}(\vec{k}-\vec{k}')P(k)$.
   (a) Define the quantity $\delta(\vec{k})$ in this expression.
   (b) Briefly explain what the angle brackets $\langle \cdots \rangle$ mean.
   (c) Show that the correlation function $\xi(r) \equiv \langle \delta(\vec{x})\delta^*(\vec{x}+\vec{r})\rangle$ is the Fourier transform of the power spectrum. *Hint:* Substitute the expression for the Fourier transform of $\delta(\vec{x})$, and then use the definition of the power spectrum from above. The angle brackets commute with the Fourier integrals.

4. **Density-velocity cross-spectrum**
   In Fourier space, the relationship between the peculiar velocity v and density contrast δ can be written as

   $$v(\vec{k}, a) = ia\frac{\dot{D}}{D}\frac{\delta(\vec{k}, a)}{k}$$

   (a) Use the definition of the growth rate, f, to rewrite this expression as $v(\vec{k}, a) = iaHf\,\delta(\vec{k}, a)/k$.
   (b) The cross-correlation between the density contrast and velocity is defined as

   $$\langle v(\vec{k}, a)\delta^*(\vec{k}', a)\rangle = (2\pi)^3\delta^{(3)}(\vec{k}-\vec{k}')P_{v\delta}(k),$$

   where $P_{v\delta}(k)$ is the *density-velocity cross-spectrum*. Show that $P_{v\delta}(k) = iaHfP(k)/k$.

(PLEASE TURN OVER)

Practice exam question: Growth rate of structure

The growth rate, f, is defined as

$$f(a) = \frac{d\log D}{d\log a}.$$

Consider a scenario in which $f = \alpha$, where α is a constant.

(a) Integrate the equation for the linear growth rate to find an expression for $D(a)$, subject to the initial condition $D = 1$ at $a = 1$.
[6 marks]

(b) By inserting an appropriate expression for the expansion rate, $H(a)$, find a simplified expression for the growth equation,

$$\frac{d}{da}\left(a^3 H(a)\frac{dD}{da}\right) = \frac{3}{2}\frac{H_0^2\Omega_m a^{-3}}{H^2(a)} a H(a) D(a),$$

under the assumption of a flat, matter-only universe.
[6 marks]

(c) Insert your solution for $D(a)$ into the simplified growth equation. Use the result to show that $\alpha = 1$ in a flat, matter-only universe.
[6 marks]

(d) There is another solution for α that satisfies the flat, matter-only growth equation. Find this value, and sketch how $D(a)$ varies with scale factor in this case.
[4 marks]

(e) Comment on how the density contrast would change with time in this case, and suggest why we usually only consider the $\alpha = 1$ solution to the growth equation.
[3 marks]