SCHOOL OF PHYSICS & ASTRONOMY *QMUL*
SPA5302 *NUCLEAR PHYSICS & ASTROPHYSICS*

END OF TERM TEST 28 NOVEMBER 2018

TIME AVAILABLE: 1 HOUR 30 MINUTES

Answer the questions in the spaces provided on these question sheets. If you run out of room for an answer, continue on the back of the page or at the end of the book.
Only non-programmable calculators may be used during this test.
This test has 13 pages of questions.
There are 60 marks available. FULL MARKS 50 = 100%

Name: ______________________________

Student number: ______________________________

Signature: ______________________________

Do not write below this line

Question	Points	Score
1	5	
2	5	
3	5	
4	5	
5	5	
6	5	
7	5	
8	5	
9	5	
10	5	
11	5	
12	5	
Total:	60	

USEFUL INFORMATION

The radius of a nuclei may be approximated by $R \approx 1.2A^{1/3}$ fm.
The semi-empirical mass formula (SEMF) for the binding energy of a nucleon is

$$B(Z,A) = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(Z,A)\,.$$

Constants in the SEMF: $a_V = 15.56, a_S = 17.23, a_C = 0.697, a_A = 23.28, a_P = 12.0$ where each number is in MeV.
Nuclear Shells: Protons

$$1s_{\frac{1}{2}} \Big\downarrow_2 1p_{\frac{3}{2}} \mid 1p_{\frac{1}{2}} \Big\downarrow_8 1d_{\frac{5}{2}} \mid 2s_{\frac{1}{2}} \mid 1d_{\frac{3}{2}} \Big\downarrow_{20} 1f_{\frac{7}{2}} \Big\downarrow_{28} 2p_{\frac{3}{2}} \mid 1f_{\frac{5}{2}} \mid 2p_{\frac{1}{2}} \mid 1g_{\frac{9}{2}} \Big\downarrow_{50} 1g_{\frac{7}{2}} \mid 2d_{\frac{5}{2}} \mid 1h_{\frac{11}{2}} \mid 2d_{\frac{3}{2}} \mid 3s_{\frac{1}{2}} \Big\downarrow_{82} 1h_{\frac{9}{2}} \mid 2f_{\frac{7}{2}}$$

Shells: Neutrons

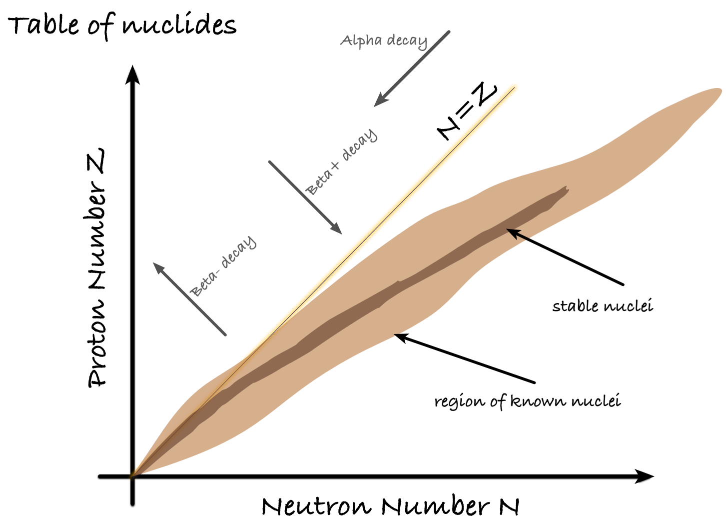
$$1s_{\frac{1}{2}} \Big\downarrow_2 1p_{\frac{3}{2}} \mid 1p_{\frac{1}{2}} \Big\downarrow_8 1d_{\frac{5}{2}} \mid 2s_{\frac{1}{2}} \mid 1d_{\frac{3}{2}} \Big\downarrow_{20} 1f_{\frac{7}{2}} \Big\downarrow_{28} 2p_{\frac{3}{2}} \mid 1f_{\frac{5}{2}} \mid 2p_{\frac{1}{2}} \mid 1g_{\frac{9}{2}} \Big\downarrow_{50} 2d_{\frac{5}{2}} \mid 1g_{\frac{7}{2}} \mid 1h_{\frac{11}{2}} \mid 2d_{\frac{3}{2}} \mid 3s_{\frac{1}{2}} \Big\downarrow_{82} 2f_{\frac{7}{2}} \mid 1h_{\frac{9}{2}}$$

$\frac{e^2}{4\pi\varepsilon_0}$	$= 1.439965\,\mathrm{MeV\,fm}$
Boltzmann's constant	$k_B = 8.6173303 \times 10^{-5}\,\mathrm{eV/K}$
Planck's constant	$h = 4.135668 \times 10^{-15}\,\mathrm{eV\,s}$
Speed of light	$c = 2.99792 \times 10^8\,\mathrm{m/s}$
Neutrino mean lifetime	$881\,\mathrm{s}$
Atomic mass unit	$1\,u = 931.4940954\,\mathrm{MeV}/c^2 = 1.66054 \times 10^{-27}\,\mathrm{kg}$
Mass of electron	$m_e = 5.4858 \times 10^{-4}\,u = 0.51099895\,\mathrm{MeV}/c^2$
Mass of proton	$m_p = 1.00727646688\,u = 938.27208\,\mathrm{MeV}/c^2$
Mass of neutron	$m_n = 1.00866491578\,u = 939.56541\,\mathrm{MeV}/c^2$
Mass of $^1_1$H	$= 1.00782503\,u$
Mass of $^2_1$H	$= 2.01410178\,u$
Mass of $^3_1$H	$= 3.01604927\,u$
Mass of $^3_2$He	$= 3.01602932\,u$
Mass of $^4_2$He	$= 4.00260325\,u$
Mass of $^{232}_{90}$Th	$= 232.038055\,u$
Mass of $^{234}_{90}$Th	$= 234.043601\,u$
Mass of $^{235}_{92}$U	$= 235.043930\,u$
Mass of $^{236}_{92}$U	$= 236.045568\,u$
Mass of $^{238}_{92}$U	$= 238.050788\,u$
Mass of $^{239}_{92}$U	$= 239.054293\,u$
Mass of $^{240}_{94}$Pu	$= 240.053811\,u$
Mass of $^{241}_{94}$Pu	$= 241.056849\,u$
Mass of $^{242}_{94}$Pu	$= 242.058741\,u$
Mass of the Sun	$M_\odot = 1.988 \times 10^{30}\,\mathrm{kg}$
Gravitational constant	$G = 6.67408 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$

Nuclei masses given are atomic masses.

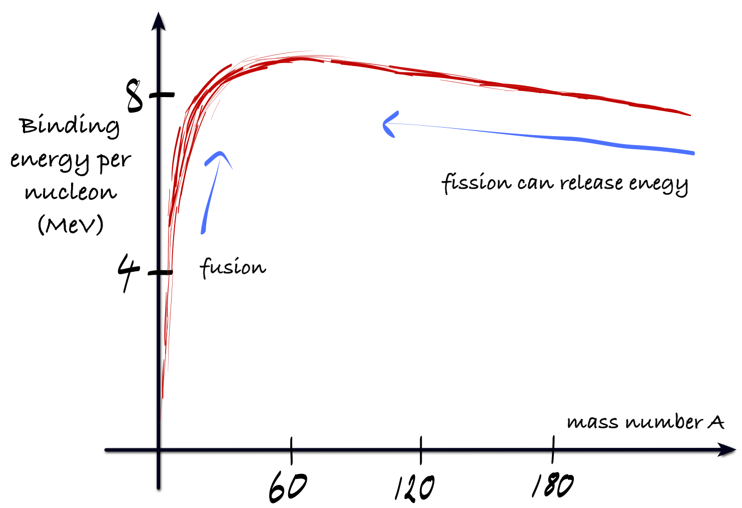
1. Sketch a table of nuclides. Indicate the region where stable nuclei are found. Indicate the direction in which β^--decay, β^+-decay, and α-decay happen. [5]

Solution:



2. Sketch a plot of binding energy per nucleon versus mass number. Indicate the regions and directions in which fission and fusion can release energy. [5]

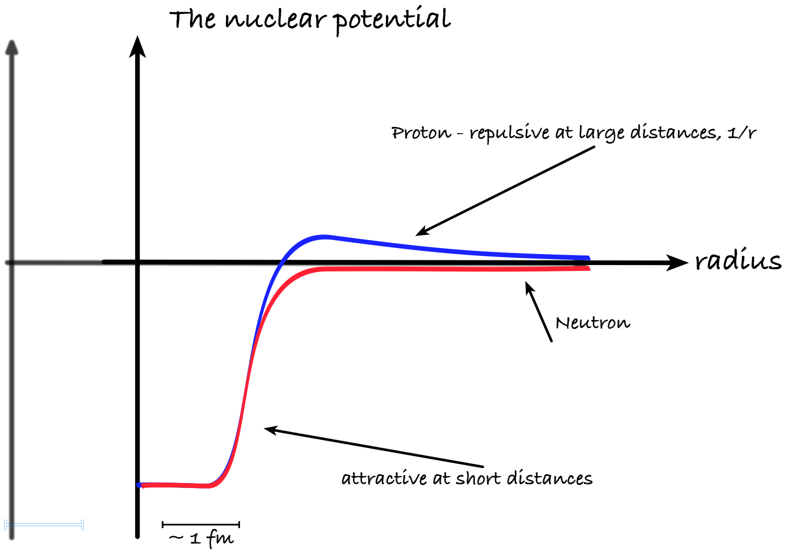
Solution:



EXTRA SPACE FOR ANSWERS

3. Sketch the nuclear potential that a proton would feel moving in the vicinity of a heavy nucleus. Explain the key features of your diagram. [5]

Solution:



4. Give the nuclear configuration of ${}^{29}_{14}$Si, and hence find the ground state J^{π}. [5]

Solution:

Protons are even so don't contribute. $N = 15$ so the neutron shell which has the last unpared nucleon is $(2s_{1/2})^1$. ground state is therefore $1/2^+$

EXTRA SPACE FOR ANSWERS

5. Starting from the Pauli exclusion principle, discuss the form of the asymmetry term in the semi-empirical mass formula. [5]

Solution:

PEP: 2 identical fermions can't occupy same state.

Each energy level can take only 2n + 2p (spin up and down). Lowest energy state when # of n and p are the same, which is more strongly bound. $(N-Z)^2$ accounts for this asymmetry - larger for more asymmetric numbers. (Arguing for the power is not necessary.) The $1/A$ factor comes from the approximate spacing between the energy levels $\sim 1/$volume of potential well.

6. Show that the semi-empirical mass formula predicts the valley of stability in the table of nuclides. Show that we expect stable heavy nuclei to have more neutrons than protons. (You may ignore the pairing term in this question.) [5]

Solution:

Let

$$B = \alpha + \beta Z - \gamma Z^2 \tag{1}$$

Where,

$$\alpha = a_v A - a_s A^{\frac{2}{3}} - a_A A, \quad \beta = 4a_A, \quad \gamma = \frac{a_c}{A^{\frac{1}{3}}} + \frac{4a_A}{A} \tag{2}$$

To find maximum binding energy, or minimum mass [1]

$$\frac{\partial B}{\partial Z} = 0 \tag{3}$$

Therefore

$$0 = \beta - 2\gamma Z \tag{4}$$

which implies, for Z_{min},

$$Z = \frac{\beta}{2\gamma} \approx \frac{4a_A}{\frac{8a_A}{A} + \frac{2a_c}{A^{\frac{1}{3}}}} \tag{5}$$

$$Z = \frac{4A}{8 + 2A^{\frac{2}{3}}\frac{a_c}{a_A}} \tag{6}$$

$$Z = \frac{A}{2}\frac{1}{1 + \frac{1}{4}\frac{a_c}{a_A}A^{\frac{2}{3}}} \tag{7}$$

[3]

Since the denominator is > 1, heavy nuclei have more neutrons [1]

EXTRA SPACE FOR ANSWERS

7. Write down the reaction equation for β^+ and β^- decay at the level of protons and neutrons. By calculating the Q-values for these reactions discuss whether free protons and neutrons are stable. [5]

Solution:

$$p \to n + e^+ + \nu_e$$

has negative Q value so p can't decay.

$$n \to p + e^- + \bar{\nu}_e + 0.782 MeV$$

$Q > 0$ so n is unstable.

8. Uranium-236 and Uranium-238 both undergo α-decay to isotopes of Thorium. By considering the Q-values for these reactions, which decay would you expect to have the longer half-life and why? [5]

Solution:

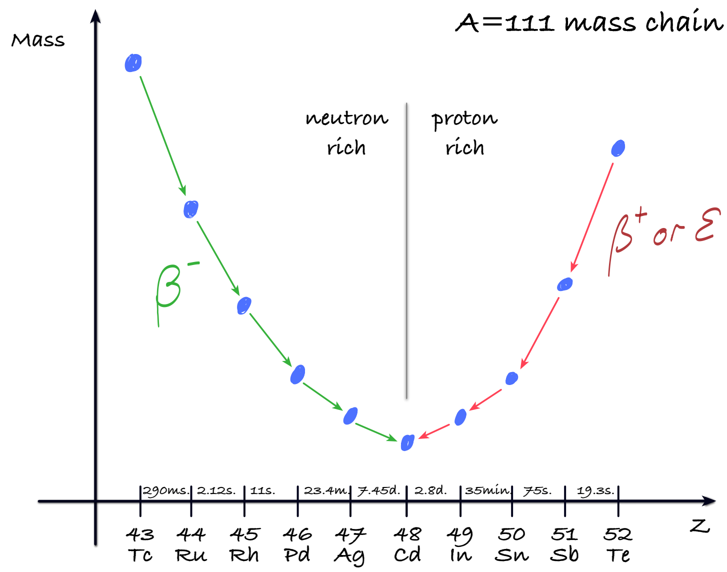
For U-238 $(238.050788 - 4.00260325 - 234.043601)931.494 = 4.269782197 MeV$
for U-236 $(236.045568 - 4.00260325 - 232.038055)931.494 = 4.573449241 MeV$

From the Geiger Nuttal rule we expect the one with the highest Q-value to have the shorter half-life, so U-236

EXTRA SPACE FOR ANSWERS

9. Explain, with the aid of a suitable diagram, how β-decay leads to the valley of stability. [5]

Solution:



10. Show that we might expect spontaneous fission to occur only in nuclei with $Z^2 \gtrsim 50A$. [5]

[You may wish to note that if a heavy nucleus is slightly deformed from a sphere to an ellipsoid, then in the SEMF the surface and Coulomb terms change, with $\epsilon \ll 1$, according to

$$E_S = a_S A^{\frac{2}{3}} \left(1 + \frac{2}{5}\epsilon^2 + ...\right), \qquad E_C = a_C Z^2 A^{-\frac{1}{3}} \left(1 - \frac{1}{5}\epsilon^2 + ...\right) . \quad]$$

Solution: So, let's deform the sphere into an ellipsoid, maintaining rotational symmetry about 1 axis for simplicity. So the change in energy is,

$$\Delta E = (E_S + E_C)_{\text{ellipse}} - (E_S + E_C)_{\text{SEMF}} \tag{8}$$

$$\simeq \frac{\epsilon^2}{5}(2a_S A^{\frac{2}{3}} - a_C Z^2 A^{-\frac{1}{3}}) \tag{9}$$

$$< 0 \quad \text{for spontaneous fission} . \tag{10}$$

Therefore,

$$\frac{Z^2}{A} \gtrsim \frac{2a_S}{a_C} \simeq 49 \tag{11}$$

$$\text{With } \frac{Z}{A} \sim 0.4 \to Z \gtrsim \frac{50}{0.4} \sim 120 . \tag{12}$$

So we see that the surface term acts to pull the shape back to a sphere, and the Coulomb term acts to break it apart.

EXTRA SPACE FOR ANSWERS

11. Calculate the Coulomb barrier for a deuteron-deuteron collision. What temperature does this correspond to for a thermal mixture? [5]

Solution:

the barrier is roughly:

$$V_c = \frac{e^2}{4\pi\epsilon_0}\frac{Z_a Z_X}{R_a + R_X} \quad (13)$$

$$\simeq 1.198\frac{Z_a Z_X}{A_a^{\frac{1}{3}} + A_X^{\frac{1}{3}}} MeV \qquad [R \approx 1.2A^{1/3}\,\text{fm}] \quad (14)$$

$$\simeq 0.15A^{\frac{5}{3}} MeV \quad \text{if} \quad A_a \simeq A_X \simeq 2Z_a \simeq 2Z_X\,. \quad (15)$$

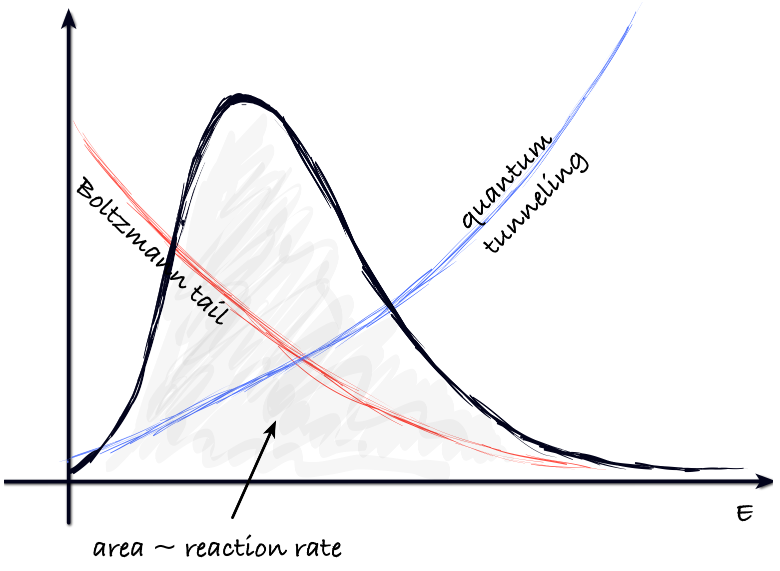
In D-D reactions this is $\simeq 0.5MeV$

$$E = k_b T\,, \quad (16)$$

where k_b is the Boltzmann Constant ($\simeq 8.62e - 11MeV/K$). This estimate means that we'd require a gas with $T \sim e10K$

12. Explain, with the aid of a suitable diagram, two processes which allow the Coulomb barrier to be overcome and fusion to occur in a hot mixture of light nuclei. [5]

Solution:



The integrand in the reaction rate is a product of the falling tail of the Boltzmann distribution and the increasing probability of quantum tunnelling. The area under the Gamow peak is proportional to the reaction rate.

EXTRA SPACE FOR ANSWERS