Revision Questions I

Week 1: Expanding universe

1. An emission line has a rest-frame wavelength of 1240 nm. The same emission line is observed in a distant galaxy at a wavelength of 1377 nm. What is the redshift of the galaxy?
2. The true cosmological redshift of a galaxy is $z = 0.014$. Its peculiar velocity is 720 km/s in the direction away from us. Calculate the *observed* redshift of the galaxy.
3. Calculate the recession velocity of a galaxy with a true cosmological redshift of $z = 0.34$ and a peculiar velocity of 402 km/s in the direction towards us.
4. Explain how the Doppler shift affects the wavelength of light. Make sure to explain the sign of the effect.
5. State Olbers' paradox. How is it resolved?
6. The expansion rate today is $H_0 \approx 70$ km/s/Mpc. If a galaxy of initial diameter 42 kpc was expanding with the Hubble flow, how much bigger would it be after 1000 years?
7. Write down an expression for the observed redshift z of an object with cosmological redshift z_c and peculiar velocity v.
8. What is the difference between the recession velocity and peculiar velocity of an object?
9. Show that typical galaxy peculiar velocities ($v \lesssim 1000$ km/s) are an important contribution to the observed redshift for nearby (low-redshift) objects, but are negligible at high redshifts.

Week 2: Geometry and distance

1. State the Hubble Law. Explain why the Hubble Law is predicted by an expanding universe.
2. What is the parallax of an object at a distance of 102 pc? Give your answer in arcsec.
3. Explain the difference between parallax angle and the angular size of an object.
4. Explain the difference between proper and comoving coordinates.
5. If an object stays at an exactly constant comoving distance from us as the Universe expands, what does this say about its peculiar velocity?
6. Write down the line element for a flat FLRW metric in spherical polar coordinates. What is the meaning of ds?
7. State the three different types of spatial curvature that a homogeneous space can have.
8. For each type of spatial curvature, state the sign of k and the sign of Ω_k. Write down inequalities for the sum of angles in a triangle and the total matter/energy density (compared to critical density) in each case.
9. Calculate the Taylor expansion of (i) $\sin x$; (ii) $\cos x$; and (iii) e^x about $x = 0$. Give your answers to linear order in the expansion.

Week 3: Friedmann equation

1. Write down the Friedmann equation.
2. Define the expansion rate H in terms of time derivatives of the scale factor.
3. Write down an expression for ρ_Λ, the energy density of the cosmological constant Λ.
4. Calculate the expansion rate at a redshift $z = 1.4$ in a flat, matter-only Universe with $H_0 = 67$ km/s/Mpc.

5. Consider a flat universe containing only matter and a cosmological constant. The expansion rate at three redshifts is given by: $H(z = 0.1) = 73.54$ km/s/Mpc; $H(z = 0.7) = 96.85$ km/s/Mpc; $H(z = 1.4) = 138.79$ km/s/Mpc. Use these data to calculate the values of Ω_Λ and H_0.
6. Write down the definition of the critical density today, $\rho_{\rm cr,0}$.
7. Write down the definition of Ω_m in terms of the critical density today and the matter density $\rho_m(t)$.
8. Divide the Friedmann equation by the critical density. Use your result to prove that $\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$ in a universe that contains matter, radiation, curvature, and a cosmological constant.
9. Define Ω_0. How is Ω_0 related to Ω_k?
10. Write down the definition of Ω_k in terms of the curvature constant, k.
11. Calculate Ω_k for a universe where $\rho_{\rm tot}(t_0) = 0.93\rho_{\rm cr,0}$.
12. Calculate the critical density of a universe with $H_0 = 102$ km/s/Mpc. State your answer in units of kg/m^3.
13. Consider a universe that is made up of 12% massless neutrinos, 28% CMB photons, and 14% cold dark matter. The remainder is made up of a cosmological constant. What is Ω_r in this universe?
14. Write down how the densities of matter, radiation, and a cosmological constant vary with scale factor, a.
15. Solve the Friedmann equation to find $a(t)$ for a flat, matter-only universe with initial conditions $a = 0$ at $t = 0$. Calculate the age of this universe, t_0, in Gyr.
16. Solve the Friedmann equation to find $a(t)$ for a flat, radiation-only universe with initial conditions $a = 0$ at $t = 0$. Sketch your solution, $a(t)$.
17. Solve the Friedmann equation to find $a(t)$ for an empty universe with negative curvature, and initial conditions $a = 0$ at $t = 0$. Sketch your solution, $a(t)$.
18. Solve the Friedmann equation to find $a(t)$ for a flat, cosmological constant-only universe with initial conditions $a = 0$ at $t = 0$. If $H_0 = 82$ km/s/Mpc in this universe, what will be the value of the scale factor when it reaches an age of 10 Gyr?
19. Explain the differing fates of a matter-only universe that is (i) flat; (ii) open; (iii) closed.
20. Find expressions for the following quantities in terms of the expansion rate, H, and scale factor, a: (i) da/dt; (ii) da/dz; (iii) dz/dt; (iv) dk/dt (where k is the curvature parameter).

Week 4: Distances and horizons

1. Write down the definition of the luminosity distance in terms of the observed flux f and intrinsic luminosity L of an object.
2. Write down the definition of the angular diameter distance of an object in terms of its angular size $\Delta\theta$ and physical (proper) diameter d.
3. Is the angle $\Delta\theta$ in the definition above in degrees or radians?
4. Explain the difference between the parallax distance and angular diameter distance.
5. Write down definitions of the luminosity distance and angular diameter distance in terms of the comoving distance travelled by light, $r(z)$, and redshift z.
6. If a galaxy at redshift $z = 3.83$ lies at an angular diameter distance of $d_A = 1460$ Mpc, what is its luminosity distance (in Mpc)? What is the comoving distance travelled by light from this galaxy?
7. A galaxy of proper (physical) diameter 41 kpc is observed to subtend an angle of 1.8 arcsec. What is the angular diameter distance to the galaxy?

8. A galaxy has an angular size of 1.5 arcsec at a redshift $z = 0.5$. Calculate its physical (proper) diameter in a matter-only universe with $H_0 = 71$ km/s/Mpc.
9. Calculate the proper (physical) diameter of a galaxy that is observed with an angular size of 4.1 arcsec at an angular diameter distance of $d_A = 104$ Mpc.
10. Explain the terms *standard candle* and *distance ladder*.
11. A nearby galaxy contains a Cepheid variable star with known luminosity $L = 2890\,L_\odot$, observed with flux $f = 4.5 \times 10^{-3}\,L_\odot/\mathrm{Mpc}^{-2}$. Calculate the luminosity distance to the galaxy.
12. Use the FLRW line element to find an integral expression for the comoving distance travelled by light in a flat universe with arbitrary expansion rate $H(a)$.
13. Derive an expression for the comoving distance travelled by light, $r(z)$, in a flat, matter-only universe.
14. Find expressions for the following quantities in terms of the expansion rate, H, and scale factor, a: (i) dr/dt; (ii) dr/da; (iii) dr/dz.
15. Define the term *particle horizon*. Write down an expression for the particle horizon in any flat FLRW universe.
16. Write down a general expression for the Hubble radius, $r_{\mathrm{HR}}(a)$.
17. Calculate the Hubble radius for (i) a matter-only universe; (ii) a radiation-only universe; (iii) a cosmological constant-only universe. You may assume flatness.
18. Sketch the Hubble radius as a function of scale factor in the three universes from the question above.

Week 5: Cosmic acceleration

1. Write down the equation of state parameters for (i) matter; (ii) radiation; (iii) a cosmological constant; (iv) massless relativistic neutrinos; (v) baryons; (vi) cold dark matter.
2. Use the conservation equation to show that a type of mass/energy with constant density ($\rho(t) = \mathrm{const.}$) has an equation of state $w = -1$.
3. Consider a flat universe with $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, and $H_0 = 60$ km/s/Mpc. Use the Raychaudhuri equation to find the scale factor at which the expansion of this universe first starts to accelerate ($\ddot{a} > 0$).
4. Show that $d(\dot{a}^2)/dt = 2\dot{a}\ddot{a}$.
5. Calculate the time derivative of $y = (\dot{a}/a)^2$.
6. Explain why the following definition cannot be correct: $q(a) = -(1 + \dot{H}/H)$.
7. The deceleration parameter is given by $q(a) = -(1 + \dot{H}/H^2)$. Show that this leads to the following alternative definition:
$$q(a) = -\left(\frac{a}{\dot{a}}\right)^2 \frac{\ddot{a}}{a}. \tag{5}$$
8. Calculate the deceleration parameter today, q_0, in (i) a matter-only universe; (ii) a cosmological constant-only universe; (iii) an empty universe with negative curvature ($k < 0$).
9. Solve the Friedmann equation to find $a(t)$ for a flat, cosmological constant-only universe. Use your solution to show that $H = \mathrm{const.}$
10. Sketch the Hubble radius as a function of scale factor in a flat, cosmological constant-only universe.
11. Brielfy explain the *cosmological constant problem*.
12. Using the Raychaudhuri equation, conservation equation, or otherwise, derive a relation between q and w.
13. Use the conservation equation to find how the density depends on scale factor for types of mass/energy with equation of state (i) $w = 0$; (ii) $w = -2/3$; and (iii) $w = +1$.